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Finite element study of acoustic waves

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FINITE ELEMENT STUDY OF ACOUSTIC WAVES

by

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THESIS

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December 1970

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ABSTRACT

The generation and propagation of small amplitude acoustic waves in a homogeneous, loss-free, compressible fluid is studied by the finite element method. A diaphragm mounted in an infinite rigid baffle generates acoustic waves in a semi-infinite fluid region. Steady-state pressure distribution is found for a hemispherical region with boundary reflection suppressed through use of a radiation condition. The computer program developed for the purpose utilizes iso-parametric finite elements with curvilinear boundaries. Incorporated in the program is a versatile mesh generator which minimizes the quantity of input data. Acceptable agreement with analytic results is obtained when there are at least four elements per wave-length.

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I. INTRODUCTION

The field of acoustics plays an important role in the development of the complex sonar systems used in the United States Navy today. It is the intent of this paper to open a new avenue to the study of the active sonar system. Attention is confined to the transducer and the periodic pressure field induced in a fluid medium by harmonic oscillations of the transducer piston or diaphragm.

The study is restricted to the steady-state performance of a transducer mounted in an infinite rigid baffle and radiating into a half-space filled with a homogeneous fluid. Both the pressure field and the acoustic impedance of the transducer are determined. Figure 1 illustrates the geometry of the region.

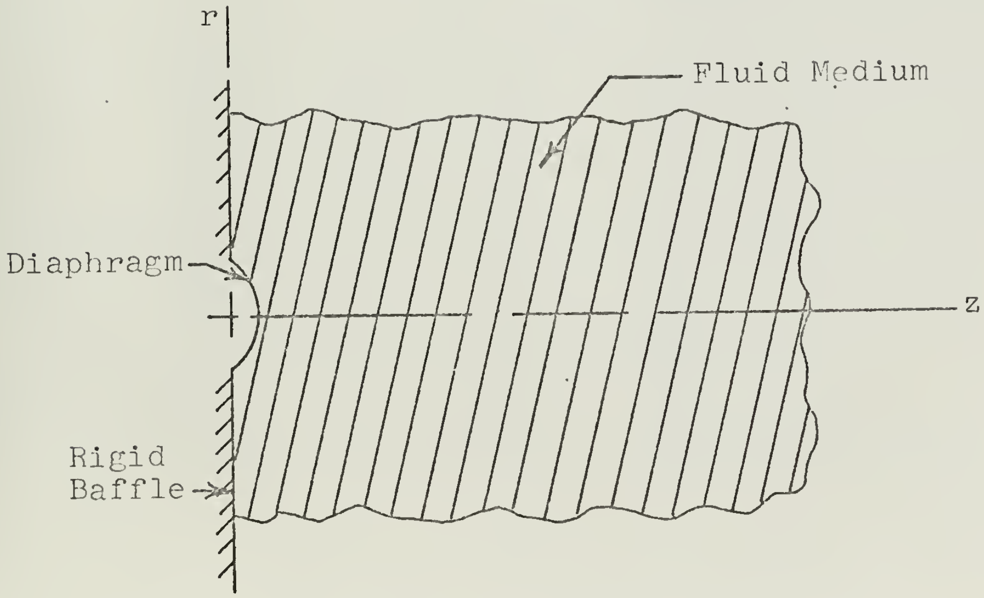

Figure 1. Geometry of Region.

The method used in this study is the two-dimensional finite element technique. Since it is manifestly impossible to represent an infinite region with a finite number of finite elements, the stratagem of the anechoic chamber is simulated mathematically. This is accomplished by using radiation (non-reflection) boundary conditions.

The diaphragm, being circular, forms a surface of revolution when deflected. Thus, the instantaneous pressure distribution has rotational symmetry about an axis through the diaphragm center and normal to the rigid baffle (z axis of Figure 1). A finite element assembly representing the region of interest may be constructed by dividing the corresponding portion of the r-z plane of Figure 1 into subareas as shown in Figure 2.

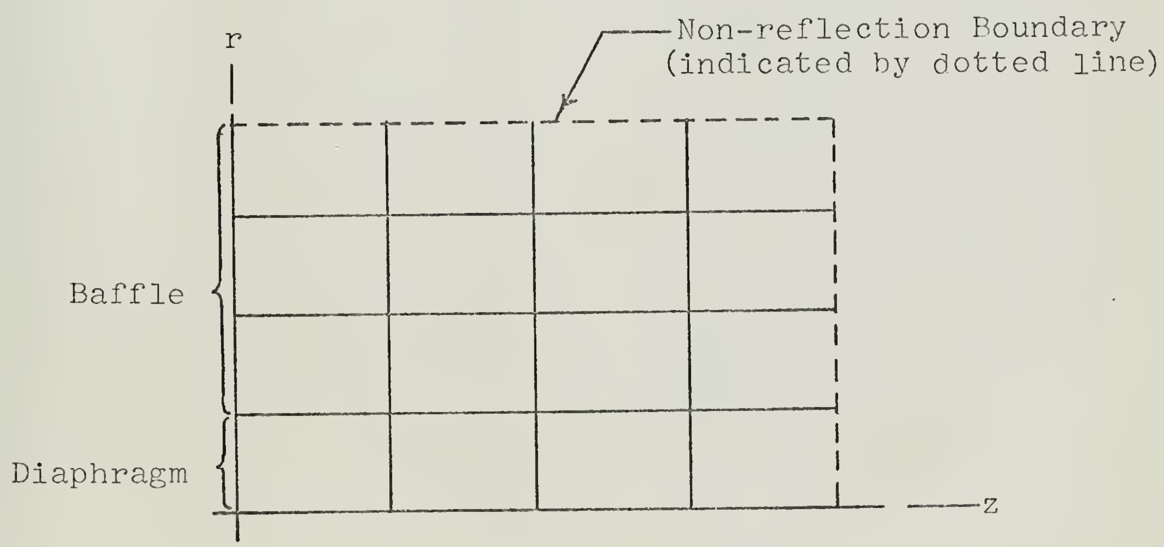

Figure 2. Finite Element Grid

In Figure 2, an individual rectangular element represents the cross-section of a toroidal element generated by revolution about the z axis. The rotational symmetry permits calculation of element properties using two-dimensional technique by simply taking the thickness (normal to the plane of the figure) proportional to r.

II. FINITE ELEMENT FORMULATION OF THE GOVERNING EQUATION

A. EQUATION OF FLUID MOTION

The governing partial differential equation for the pressure p in a homogeneous, inviscid, compressible fluid as used in acoustics (e.g., see Ref. [1]*) is

$$\nabla^2 p = \frac{1}{c^2}\ddot{p} \tag{1}$$

where ∇^2 is the Laplacian operator, c is the acoustic velocity, and superior dots represent differentiation with respect to time. For the purpose of the present study, boundary conditions take the form

$$\frac{\partial p}{\partial n} = -\rho \dot{v}_n \tag{2}$$

where the coordinate n is measured along the normal to the boundary surface, ρ is the fluid density, and $\dot{v}_n$ is the component of fluid particle velocity in the n direction.

B. SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

The finite element method replaces Eqs. (1) and (2) by a set of simultaneous ordinary differential equations with independent variable time and dependent variables consisting of pressures at a large number of discrete points (nodes) of the region. The region is divided into a number of

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*Numbers in brackets designate references listed on page 68.

sub-regions called "elements." Within each element it is postulated that the pressure p may be expressed in terms of the pressure p_i at the n nodes of the element. This relation is expressed as

$$p = \sum_{i=1}^{n} N_i p_i \tag{3}$$

where the "shape" (or interpolation) functions N_i are functions of the spatial coordinates. If the nodes are located at suitably chosen points on the element surface, unique sets of shape functions will guarantee the necessary continuity of pressure p along interelement boundaries as pointed out by Zienkiewicz [2]. The elements used in this study are two-dimensional eight-noded iso-parametric elements. Further details are given in Appendix C.

It has been shown by Zienkiewicz and Newton [3] that the finite element equation for k nodes, m of which are located on the fluid-diaphragm interface, may be written as

$$[Q]\{\ddot{p}\} + [D]\{\dot{p}\} + [H]\{p\} = -\rho[L]^T\{\ddot{\delta}\} \tag{4}$$

where {p} is a k x 1 vector of nodal pressures, [Q], [D], and [H] are k x k symmetric matrices, $\{\delta\}$ is an m x 1 vector of diaphragm displacements, and $[L]^T$ is a k x m matrix. Elements of [Q], [D], [H], and $[L]^T$ are all real constants. Formulas for calculating these matrices are given in Appendix E. The terms $[D]\{\dot{p}\}$ and $[L]^T\{\ddot{\delta}\}$ represent boundary conditions and will be discussed in following sections.

C. DIAPHRAGM BOUNDARY CONDITION

The diaphragm displacement δ is a prescribed function of time. It can be represented by choosing a number of points (nodes) on the diaphragm and using a set of shape functions N_j' to interpolate the local displacement as

$$\delta = \sum_{j=1}^{m} N_j' \delta_j \tag{5}$$

where the δ_j are the displacements at the m diaphragm nodes.

The boundary condition of Eq. (2) is formulated in terms of the normal component v_n of fluid particle velocity. At the diaphragm surface this must, in the absence of cavitation, be equal to $\dot{\delta}$. Using this equality, the fluid-diaphragm interface condition gives the term

$$-\rho [L]^T \{\ddot{\delta}\}$$

in the finite element equation (Eq. (4)). The resulting vector has nonzero elements only for those pressure nodes located on the diaphragm surface. It is noted that the boundary condition on the rigid baffle is a special case, in that the particle velocity $v_n = 0$, which implies $\dot{v}_n = 0$.

D. RADIATION (NON-REFLECTION) BOUNDARY CONDITION

A normally incident wave meeting the interface between two media will not be reflected if the specific acoustic impedances of the two media are equal. The specific acoustic impedance z is defined to be the ratio of pressure to

particle velocity. For plane harmonic waves this is a real quantity and the pressure can be given as

$$p = \rho c v_n \tag{6}$$

for a wave progressing in the positive n direction. Differentiating Eq. (6) with respect to time and using this result to eliminate $\dot{v}_n$ from the boundary condition (Eq. (2)) gives

$$\frac{\partial p}{\partial n} = - \frac{1}{c} \dot{p} \tag{7}$$

Equation (7) then becomes the boundary condition for non-reflection of normally incident plane waves. The term

$$[D]\{\dot{p}\}$$

in Eq. (4) results from this criterion. This vector has nonzero elements only for those nodes which lie on the non-reflecting portion of the boundary.

III. GEOMETRY OF THE REGION

The initial solution of the problem using the geometric configuration of Figure 2 resulted in significant error in the pressure distribution when the solution was checked against known results. The principal cause of this error was believed to be the shape of the non-reflection boundary. By the nature of the geometry, the harmonic oscillating diaphragm will create progressive pressure waves which propagate outward and strike the boundary of Figure 2 at various angles of incidence. The particle velocity, v_n, will not always be normal to the non-reflection boundary and the boundary condition required by Eq. (2) will not be satisfied. Thus, a modification to the representation of the boundary to provide substantially normal incidence led to the geometry illustrated in Figure 3. For this modified radiation boundary, in three dimensions the surface is hemispherical.

In order to represent adequately the circular arc of the non-reflection boundary, two-dimensional iso-parametric finite elements are used. The element thickness normal to the plane of the figure is directly proportional to the radius r. Iso-parametric elements are further discussed in Appendix C.

Note the distortion of the grid in Figure 3. The reason for this distortion and the method of generating the grid

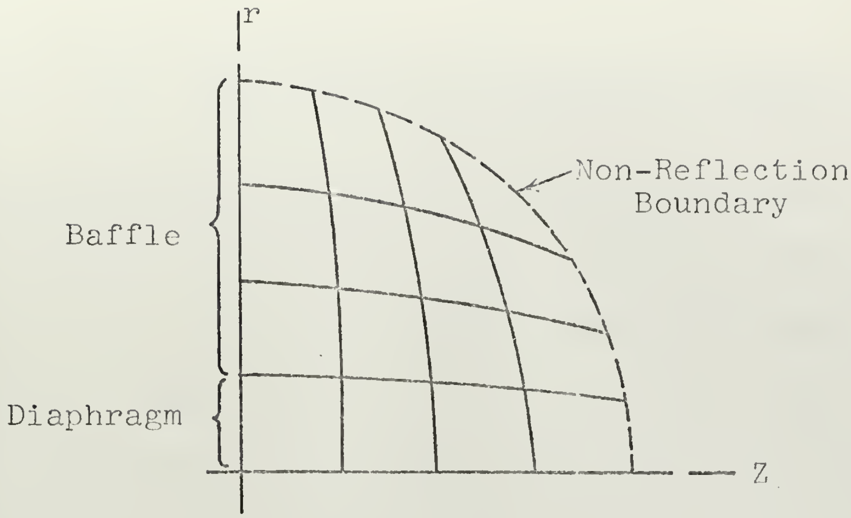

Figure 3. Modified Finite Element Region

pattern is discussed in Appendix D. The distorted "rectangles" still represent the cross sections of toroidal elements.

IV. WAVE THEORY AT RADIATION BOUNDARY

A one-dimensional study of a pulsating sphere acting on a fluid medium resulted in a more accurate solution when spherical wave theory at the non-reflection boundary was introduced. Based upon the success of these results, a further modification was made to the computer program (Appendix B) to incorporate spherical wave theory in the two-dimensional study. This was accomplished by developing the non-reflection boundary condition for spherically diverging waves as shown below.

The specific acoustic impedance for spherically diverging waves is given in Ref. [1] as

$$Z = \frac{p}{v_n} = \rho c \frac{\frac{\omega}{c} r}{\frac{\omega}{c} r - j} \tag{8}$$

where ω is the circular frequency and r is the radial distance from the "center." The particle acceleration $\dot{v}_n$ can be written as

$$\dot{v}_n = j\omega v_n \tag{9}$$

which allows the boundary condition to be represented by

$$\frac{\partial p}{\partial r} = -\rho j\omega v_n \tag{10}$$

From Eqs. (8) and (10), the boundary condition is

$$\frac{\partial p}{\partial r} = - \frac{j\omega p}{c}\left[\frac{\frac{\omega}{c}r - j}{\frac{\omega}{c}r}\right] \tag{11}$$

or

$$\frac{\partial p}{\partial r} = - p\left(\frac{1}{r} + j\frac{\omega}{c}\right) \tag{12}$$

The imaginary term on the right hand side of Eq. (12) is represented by the term $[D]\{\dot{p}\} = j\omega[D]\{p\}$ in Eq. (4). Thus, the contribution of the real term can be written as

$$\frac{c}{r}[D]\{p\} \tag{13}$$

and can be added to the system of equations given in Eq. (4).

The spherical contribution is only added to the nodes which lie on the non-reflection boundary, thus providing a spherical wave approximation to the plane wave theory at the non-reflection boundary.

V. COMPUTER SOLUTION TECHNIQUE

Assume the diaphragm is oscillating with a harmonic displacement represented in the complex form by

$$\{\delta\} = \{\delta_o\}e^{j\omega t} \tag{14}$$

where $\{\delta_o\}$ is a vector of prescribed displacement amplitudes at the m nodes of the diaphragm-fluid interface. Elements of $\{\delta_o\}$ are real. The pressure in the fluid medium is then represented by

$$\{p\} = \{p_o\}e^{j\omega t} \tag{15}$$

where $\{p_o\}$ is a k x 1 vector of pressure amplitudes (complex) at the k nodes of the region.

Inserting Eqs. (14) and (1!) into Eq. (4) gives

$$([H] - \omega^2[Q] + j\omega[D])\{p\} = \omega^2\{B\} \tag{16}$$

where $\{B\} = [L]^T\{\delta_o\}$.

In the computer program, the system matrices [H], [Q], [D], and {B} are assembled from the contributions of each individual finite element in various subroutines. Since only a small number of nodes are located on the non-reflection boundary, the matrix [D] is sparse. In order to reduce computer storage requirements, [D], which is symmetric, is stored as three vectors: the principal diagonal and the two diagonals immediately above. At the same time, the approximated spherical contribution $\frac{c}{r}[D]$ at the

non-reflection boundary is computed. These quantities are real and are added directly to the matrix [H] in the appropriate locations.

The final subroutine introduces the value of omega and multiplies the matrices [Q], [D], and {B} by the appropriate power of omega before the left-hand side is summed to form one complex matrix. Thus the system of equations is reduced to

$$[S]\{p\} = \{F\} \tag{17}$$

where [S] is a k x k complex matrix and {F} is a column vector of order k x 1 in which only m elements are nonzero.

The complex pressure amplitudes in Eq. (17) are found by using the Gaussian elimination and back substitution method.

The absolute value of the pressure and the phase angle are calculated in the program and printed out in array form. Additionally, a system energy balance and the diaphragm acoustic impedance are determined.

The program provides for multiple-problem solution by using incremented values of omega so that a range of frequencies can be studied.

VI. RESULTS

Problems studied were chosen to verify the accuracy of the method through comparison with available, exact (analytic) solutions. Results for each problem are discussed below.

Problem 1. Piston in a Rigid Tube

To verify the correctness of the computer program, finite element solutions were obtained for the problem of a harmonically oscillating circular piston at one end of a fluid-filled, rigid pipe of equal diameter. An infinite length was simulated by terminating the finite element region with a plane non-reflection boundary at a distance of one piston diameter. This is, of course, a one dimensional problem with uniform pressure amplitude throughout and phase angle is a linear function of distance along the pipe.

Detailed results are not presented for this essentially trivial test problem. It was found, however, that there was excellent agreement with theory for long wave-lengths. For wave-lengths less than approximately four times the element width (measured parallel to the direction of wave propagation), the accuracy deteriorated. This is understandable, since the shape functions employed represent the instantaneous pressure profile across an element by a parabolic arc. The lower limit on wave-length corresponds,

for a given element grid, to an upper limit on frequency.

Problem 2. Piston in an Infinite Baffle

The problem explored most extensively is that of a harmonically oscillating circular piston surrounded by an infinite rigid baffle and radiating into a semi-infinite fluid region. For this problem, the radiation boundary is a hemispherical surface with the center located at the piston center and the radius is four times that of the piston. The element grid (see Figure 4, Appendix A) contains 16 elements. It is derived by mapping a 4 x 4 square array from the ξ,η plane into a quadrant of a circle.

Analytic results for this problem are readily available (e.g., see [1], [4]). In Table I, the pressure amplitudes and phase angles along the z axis, obtained by the finite element method (FEM), are compared with exact results. Two sets of finite element results are given. For those designated FEM1, the spherical wave correction at the radiation boundary is included. For FEM2, the correction is omitted. Details are given in Table I for two circular frequencies, ω = .1 and .6 rad/sec. The corresponding wave-lengths are approximately 63 ft and 10 feet.

Examination of Table I shows that FEM1 results give generally good agreement with exact results. Those for FEM2 are considerably poorer, although the discrepancies are smaller for ω = .6 rad/sec. This is understandable since the 1/r contribution to specific acoustic impedance is relatively less important as ω increases (see Eq. (12)).

It is believed that a more refined finite element mesh (more elements) should afford significant improvement. Since the computer program for this problem required 320K bytes of core storage, it was not feasible to investigate this without major program revisions.

In predicting transducer performance the resultant fluid reaction on the piston is needed. This is normally reported in the form of piston radiation impedance, which is complex and is found by dividing the piston force by the piston velocity. The real part of the radiation impedance is called the radiation resistance and the imaginary part is called the radiation reactance.

Table II provides a comparison with exact values for the radiation resistance and reactance. Results are given for FEM1 and FEM2 over a range of frequencies. Again it is observed that the spherical correction significantly improves the results. Note that the radiation resistance obtained by FEM1 continues to give good results through $\omega = .9$ rad/sec.

Computer solutions of this problem were obtained for a range of frequency from $\omega = .1$ rad/sec to $\omega = 1.2$ rad/sec by .1 rad/sec increments. Comparisons at the upper end of this range are omitted from Tables I and II because the departure from exact results does not permit useful employment of the finite element method without grid refinement. Complete computer results for pressure amplitude, phase

TABLE I

PRESSURE AMPLITUDE AND PHASE ANGLE ON AXIS

ω rad/sec	z ft	PRESSURE AMPLITUDE EXACT lb/ft^2	FEM1 lb/ft^2	FEM2 lb/ft^2	PHASE ANGLE EXACT deg	FEM1 deg	FEM2 deg
.1	0	.01997	.02061	.01794	-5.7	-5.7	-11.6
.1	2	.00828	.00889	.00676	-13.8	-13.1	-31.9
.1	4	.00472	.00469	.00376	-24.3	-24.9	-68.3
.1	6	.00325	.00329	.00337	-35.3	-35.8	-92.1
.1	8	.00246	.00247	.00334	-46.5	-47.4	-107.3
.6	0	.678	.699	.625	-34.4	-33.2	-34.7
.6	2	.295	.300	.273	-83.0	-78.7	-89.2
.6	4	.169	.172	.186	-145.6	-145.4	-150.5
.6	6	.117	.116	.106	-211.8	-211.0	-210.6
.6	8	.089	.086	.084	-279.3	-279.2	-289.3

TABLE II

PISTON RADIATION RESISTANCE AND REACTANCE

ω rad/sec	RESISTANCE EXACT $\frac{lb.sec}{ft}$	FEM1 $\frac{lb.sec}{ft}$	FEM2 $\frac{lb.sec}{ft}$	REACTANCE EXACT $\frac{lb.sec}{ft}$	FEM1 $\frac{lb.sec}{ft}$	FEM2 $\frac{lb.sec}{ft}$
.1	.249	.254	.451	2.11	2.06	1.68
.2	.979	.978	.687	4.09	3.97	3.56
.3	2.13	2.12	1.43	5.81	5.65	5.83
.4	3.61	3.62	3.75	7.18	6.99	8.11
.5	5.32	5.34	6.75	8.13	7.87	7.43
.6	7.12	7.12	6.61	8.62	8.31	7.00
.7	8.89	8.87	7.50	8.67	8.35	8.72
.8	10.51	10.53	10.89	8.32	7.99	9.59
.9	11.90	11.97	13.60	7.64	7.24	6.69

FEM1 - Finite element results with spherical correction.
FEM2 - Finite element results without spherical correction.

Phase angle given is for pressure relative to piston displacement.

Fluid density = 1 $lb.sec/ft^4$ Piston Radius = 2 ft.

Acoustic Velocity = 1 ft/sec. Piston Amplitude = 1 ft.

angle, and impedance components at several frequencies are given in Appendix A.

Problem 3. Parabolic Diaphragm in an Infinite Baffle

This problem, a modification of the previous problem, replaces the piston by a circular diaphragm having a fixed edge and having a deflection surface in the form of a paraboloid of revolution. Study of this problem was undertaken in an effort to improve agreement in the energy balance check (discussed below) incorporated into the program. It was believed that elimination of fluid velocity discontinuity at the edge of the piston in problem 1 might result in a significant improvement.

Analytic results for this problem are not available in standard acoustic texts, but conventional techniques can be utilized to obtain the pressure p along the z axis in complex form as

$$p = \frac{2j\rho c^3 \delta_o}{\omega a^2}\left[\left(1+j\,\frac{\omega s}{c}\right)e^{-j\frac{\omega s}{c}} - \left(1+\frac{\omega^2 a^2}{2c^2}+j\,\frac{\omega z}{c}\right)e^{-j\frac{\omega z}{c}}\right]e^{j\omega t} \qquad (18)$$

where

δ_o = displacement amplitude at diaphragm center

a = diaphragm diameter

s = "slant" distance = $(z^2 + a^2)^{\frac{1}{2}}$

For this problem only a single finite element solution (FEM3) was obtained. The spherical correction is included. Table III gives a comparison with exact results (Eq. (18))

TABLE III

PRESSURE AMPLITUDE AND PHASE ANGLE ON AXIS

ω rad/sec	z ft	PRESSURE AMPLITUDE EXACT lb/ft^2	PRESSURE AMPLITUDE FEM3 lb/ft^2	PHASE ANGLE EXACT deg	PHASE ANGLE FEM3 deg
.1	0	.01322	.01427	-4.3	-4.1
.1	2	.00438	.00461	-13.1	-12.6
.1	4	.00240	.00242	-23.7	-24.2
.1	6	.00164	.00166	-34.8	-35.5
.1	8	.00124	.00125	-46.2	-47.1
.6	0	.4598	.4934	-25.7	-23.8
.6	2	.1565	.1590	-78.5	-75.5
.6	4	.0864	.0867	-142.97	-142.3
.6	6	.0589	.0584	-209.4	-209.4
.6	8	.0445	.0436	-277.8	-278.1

FEM3 - Finite element results with spherical correction.
Phase angle is for pressure relative to diaphragm displacement.
Fluid density = 1 $lb.sec/ft^4$.
Acoustic velocity = 1 ft/sec.
Diaphragm radius = 2 ft.
Diaphragm amplitude (center) = 1 ft.

for pressure amplitudes and phase angles for values of $\omega = .1$ and .6 rad/sec.

Examination of Table III shows that agreement near the diaphragm (small values of z) is not as good as FEM1 in Table I. At the radiation boundary however, the FEM3 errors are comparable to those for FEM1.

Energy Balance Check

Prior experience, by others, suggested that an energy balance check would be a useful feature of a finite element program. Specifically, since solutions represent steady-state behavior, the input power, averaged over a cycle, should equal the corresponding average value of the output power. Input occurs at the piston, or diaphragm, and output at the radiation boundary. Expressions for these quantities are derived and incorporated in the computer program.

A number of program errors in the energy balance calculations (Appendix B) prevented valid results until the final stage of the investigation. Thus, the values of average power in and average power out are not included in the computer output (Appendix A). Recent computations, with program errors corrected, have shown excellent agreement of these quantities to at least six significant digits in the lower range of the circular frequency. Even at higher frequencies, where solution accuracy (pressure amplitudes and phase angles) deteriorates markedly, five significant digit agreement is maintained.

VII. CONCLUSIONS

It has been shown that the finite element method can provide acceptable results concerning generation and propagation of acoustic waves. The parabolic iso-parametric element provides an economical, accurate means of representing regions having curved boundaries.

The principal limitation of the method is evident from the empirically deduced results that, for acceptable accuracy, a single element must not span more than one-quarter wave-length. For wave-lengths of the order of magnitude of the piston radius, the number of nodes required would be around 900. For the present program, which already utilizes 320K bytes of core storage for 65 nodes, this is far beyond capability.

APPENDIX A

COMPUTER OUTPUT

The rows of pressures and phase angles shown in the computer output correspond to the numbering system shown on the finite element region, Figure 4.

The zeros printed in the array of pressures and phase angles do not indicate computed values, but facilitate printing the output in array form.

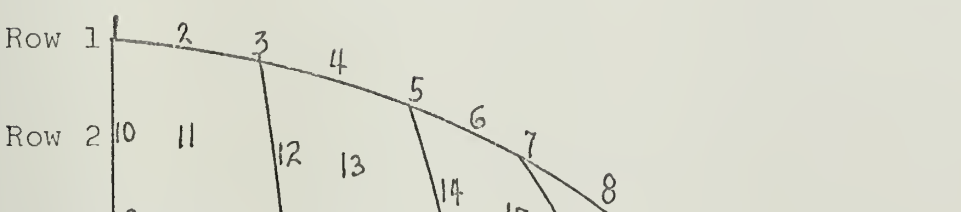

Figure 4. Format for Pressure and Phase Angle Array.

The output data are arranged according to the following sequence.

FEM1. Piston in infinite baffle with spherical correction added at the non-reflection boundary.

FEM2. Piston in infinite baffle without spherical correction added at the non-reflection boundary.

FEM3. Diaphragm in infinite baffle with spherical correction added at the non-reflection boundary.

COMPUTER OUTPUT-GENERAL

```
TITLE OF GRAPH OF CORNER NODE POINTS TO BE PLOTTED
   GRAPH ILLUSTRATING SYSTEM CORNER NODE LOCATIONS
   FINITE ELEMENT STUDY OF ACOUSTIC WAVES...D.DEAN

NUMBER OF ELEMENTS=16
NUMBER OF NODES PER ELEMENT INCLUDING CENTER NODES= 9
NUMBER OF TOTAL SYSTEM NODES INCLUDING CENTER NODES=81
NUMBER OF NODES PER ELEMENT EXCLUDING CENTER NODES= 8
NUMBER OF SYSTEM NODES EXCLUDING CENTER NODES=65
```

MAIN NODE NO	X COORD	Y COORD
1	0.0	0.0
2	4.00000	0.0
3	8.00000	0.0
4	7.39103	3.06146
5	5.65685	5.65685
6	3.06146	7.39103
7	0.0	8.00000
8	0.0	4.00000

```
NUMBER OF ELEMENT DIVISIONS IN X DIRECTION=  4
NUMBER OF ELEMENT DIVISIONS IN THE Y DIRECTION=  4
ASSIGNMENT OF NODE NUMBERS TO ELEMENTS
```

ELEMENT NO	NODE NUMBERS (8 NODED ELEMENTS)							
1	15	16	17	11	3	2	1	10
2	17	18	19	12	5	4	3	11
3	19	20	21	13	7	6	5	12
4	21	22	23	14	9	8	7	13
5	29	30	31	25	17	16	15	24
6	31	32	33	26	19	18	17	25
7	33	34	35	27	21	20	19	26
8	35	36	37	28	23	22	21	27
9	43	44	45	39	31	30	29	38
10	45	46	47	40	33	32	31	39
11	47	48	49	41	35	34	33	40
12	49	50	51	42	37	36	35	41
13	57	58	59	53	45	44	43	52
14	59	60	61	54	47	46	45	53
15	61	62	63	55	49	48	47	54
16	63	64	65	56	51	50	49	55

COMPUTER OUTPUT-GENERAL

VALUES OF XI AND ETA FOR WHICH X AND Y COORDINATES WILL BE CALCULATED

	VALUES OF XI	VALUES OF ETA
1	-1.00000	1.00000
2	-0.75000	0.75000
3	-0.50000	0.50000
4	-0.25000	0.25000
5	0.0	0.0
6	0.25000	-0.25000
7	0.50000	-0.50000
8	0.75000	-0.75000
9	1.00000	-1.00000

VALUES OF X AND Y

(X COORDINATES ON TOP-Y COORDINATES ON BOTTOM)

1	2	3	4	5	6	7	8	9
0.0 8.00000	0.80906 7.95325	1.58899 7.83617	2.33979 7.64876	3.06146 7.39103	3.75400 7.06297	4.41741 6.66459	5.05170 6.19588	5.65685 5.65685
0.0 7.00000	0.86369 6.97183	1.70190 6.88213	2.51462 6.73090	3.30185 6.51813	4.06359 6.24382	4.79984 5.90798	5.51061 5.51061	6.19588 5.05170
0.0 6.00000	0.90954 5.98678	1.79723 5.92082	2.66307 5.80211	3.50707 5.63066	4.32922 5.40647	5.12953 5.12953	5.90798 4.79984	6.66459 4.41741
0.0 5.00000	0.94659 4.99809	1.87498 4.95222	2.78516 4.86240	3.67713 4.72863	4.55090 4.55090	5.40647 4.32922	6.24382 4.06359	7.06297 3.75400
0.0 4.00000	0.97486 4.00575	1.93515 3.97634	2.88087 3.91177	3.81203 3.81203	4.72863 3.67713	5.63066 3.50707	6.51813 3.30185	7.39103 3.06146
0.0 3.00000	0.99433 3.00978	1.97773 2.99318	2.95021 2.95021	3.91177 2.88087	4.86240 2.78516	5.80211 2.66307	6.73090 2.51462	7.64876 2.33979
0.0 2.00000	1.00501 2.01016	2.00274 2.00274	2.99318 1.97773	3.97634 1.93515	4.95222 1.87498	5.92082 1.79723	6.88213 1.70190	7.83617 1.58899
0.0 1.00000	1.00690 1.00690	2.01016 1.00501	3.00978 0.99433	4.00575 0.97486	4.99809 0.94659	5.98678 0.90954	6.97183 0.86369	7.95325 0.80906
0.0 0.0	1.00000 0.0	2.00000 0.0	3.00000 0.0	4.00000 0.0	5.00000 0.0	6.00000 0.0	7.00000 0.0	8.00000 0.0

COMPUTER OUTPUT-GENERAL

```
NUMBER OF NODES ON NONREFLECTION BOUNDARY=17
NONREFLECTION BOUNDARY ALONG TOP OF ELEMENT IS FROM   XI=-1.00 TO XI= 1.00
NONREFLECTION BOUNDARY ALONG RIGHT SIDE OF ELEMENT IS FROM ETA= 1.00 TO ETA=-1.00
NODE NUMBERS LOCATED ON THE NONREFLECTION BOUNDARY
     1            7            37
     2            8            42
     3            9            51
     4           14            56
     5           23            65
     6           28
SPEED OF SOUND =   1.00000
DENSITY OF FLUID=   1.00000
NUMBER OF NODES IN CONTACT WITH PISTON=   3
RANGE ON LEFT BOUNDARY WHICH PISTON IS IN CONTACT IS FROM ETA=-0.50 TO ETA=-1.00
NODE NUMBERS IN CONTACT  WITH FORCING FUNCTION
    43
    52
    57
AMPLITUDE OF PISTON AT THE INTERFACE NODES
   NODE      AMPLITUDE
    43        1.00000000
    52        1.00000000
    57        1.00000000
INITIAL VALUE OF OMEGA=   0.10000
INCREMENT VALUE OF OMEGA=   0.10000
NUMBER OF INCREMENTS OF OMEGA=12
```

FEM1. Piston in Infinite Baffle with Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.10000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.00253 -45.990	0.00254 -45.842	0.00253 -46.006	0.00253 -46.017	0.00252 -46.136	0.00252 -46.214	0.00251 -46.317	0.00251 -46.436	0.00250 -46.628
0.00290 -40.088	0.0 0.0	0.00286 -40.722	0.0 0.0	0.00276 -42.216	0.0 0.0	0.00263 -44.241	0.0 0.0	0.00250 -46.718
0.00341 -34.109	0.00334 -34.805	0.00328 -35.467	0.00316 -36.865	0.00302 -38.494	0.00289 -40.294	0.00275 -42.368	0.00262 -44.529	0.00249 -46.870
0.00405 -28.729	0.0 0.0	0.00380 -30.605	0.0 0.0	0.00334 -34.891	0.0 0.0	0.00287 -40.608	0.0 0.0	0.00249 -47.016
0.00509 -22.841	0.00501 -23.195	0.00451 -25.837	0.00411 -28.369	0.00369 -31.626	0.00331 -35.257	0.00299 -39.147	0.00272 -43.076	0.00249 -47.123
0.00733 -15.824	0.0 0.0	0.00565 -20.573	0.0 0.0	0.00403 -28.939	0.0 0.0	0.00309 -37.837	0.0 0.0	0.00249 -47.234
0.01232 -9.420	0.00907 -12.820	0.00669 -17.413	0.00532 -21.916	0.00437 -26.705	0.00369 -31.724	0.00318 -36.860	0.00280 -42.074	0.00250 -47.289
0.01858 -6.260	0.0 0.0	0.00777 -15.019	0.0 0.0	0.00463 -25.239	0.0 0.0	0.00324 -36.232	0.0 0.0	0.00251 -47.320
0.02061 -5.651	0.01175 -9.930	0.00889 -13.111	0.00621 -18.791	0.00469 -24.919	0.00385 -30.488	0.00329 -35.746	0.00282 -41.573	0.00247 -47.442

ACOUSTIC IMPEDANCE

REAL= 0.25423472

IMAG= 2.05667215

FEM1. Piston in Infinite Baffle with Shperical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.30000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.02178 -136.437	0.02174 -136.276	0.02177 -136.498	0.02174 -136.673	0.02174 -136.943	0.02174 -137.175	0.02175 -137.462	0.02176 -137.781	0.02177 -138.320
0.02481 -119.026	0.0 0.0	0.02450 -120.845	0.0 0.0	0.02378 -125.201	0.0 0.0	0.02284 -131.196	0.0 0.0	0.02180 -138.559
0.02897 -101.218	0.02876 -102.948	0.02808 -105.131	0.02722 -109.152	0.02620 -113.958	0.02508 -119.366	0.02398 -125.519	0.02287 -131.990	0.02181 -138.987
0.03496 -84.668	0.0 0.0	0.03290 -90.291	0.0 0.0	0.02897 -103.182	0.0 0.0	0.02509 -120.241	0.0 0.0	0.02184 -139.374
0.04404 -67.107	0.04289 -68.438	0.03933 -75.919	0.03572 -83.632	0.03213 -93.411	0.02893 -104.217	0.02620 -115.755	0.02384 -127.552	0.02184 -139.704
0.06255 -46.539	0.0 0.0	0.04889 -60.614	0.0 0.0	0.03547 -85.250	0.0 0.0	0.02719 -111.874	0.0 0.0	0.02194 -139.998
0.10619 -27.507	0.07915 -37.559	0.05916 -51.048	0.04714 -64.366	0.03866 -78.518	0.03255 -93.566	0.02803 -108.899	0.02462 -124.498	0.02203 -140.164
0.16362 -18.278	0.0 0.0	0.06930 -44.035	0.0 0.0	0.04110 -74.283	0.0 0.0	0.02862 -106.979	0.0 0.0	0.02212 -140.299
0.18304 -16.532	0.10585 -28.998	0.07853 -38.569	0.05498 -55.329	0.04205 -73.054	0.03434 -89.622	0.02888 -105.764	0.02486 -123.098	0.02181 -140.440

ACOUSTIC IMPEDANCE

REAL= 2.11780205

IMAG= 5.65370620

FEM1. Piston in Infinite Baffle with spherical Correction
Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.60000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.07559 86.954	0.07613 86.875	0.07617 86.955	0.07693 86.417	0.07759 86.167	0.07835 85.708	0.07948 85.323	0.08040 84.812	0.08172 83.943
0.08637 121.203	0.0 0.0	0.08608 117.825	0.0 0.0	0.08540 109.550	0.0 0.0	0.08432 97.978	0.0 0.0	0.08282 83.676
0.09955 156.262	0.10061 154.261	0.09853 149.116	0.09735 141.666	0.09547 132.479	0.09262 121.592	0.09021 109.661	0.08687 96.811	0.08416 83.067
0.12293 -168.583	0.0 0.0	0.11745 179.914	0.0 0.0	0.10695 153.794	0.0 0.0	0.09566 120.244	0.0 0.0	0.08505 82.491
0.15587 -133.372	0.14923 -137.699	0.14403 -150.744	0.13192 -167.172	0.12155 173.329	0.11097 152.061	0.10184 129.651	0.09312 106.141	0.08592 81.974
0.21095 -93.934	0.0 0.0	0.17615 -122.376	0.0 0.0	0.13770 -170.201	0.0 0.0	0.10732 137.294	0.0 0.0	0.08690 81.489
0.36675 -54.593	0.28723 -75.328	0.22673 -102.174	0.18449 -128.915	0.15359 -157.248	0.13028 173.449	0.11200 143.391	0.09811 112.580	0.08750 81.173
0.60717 -36.413	0.0 0.0	0.27351 -88.602	0.0 0.0	0.16594 -148.976	0.0 0.0	0.11558 147.324	0.0 0.0	0.08829 80.855
0.69899 -33.217	0.42606 -57.874	0.30000 -78.708	0.21718 -112.362	0.17233 -145.348	0.14117 -177.879	0.11589 148.974	0.09995 115.227	0.08645 80.824

ACOUSTIC IMPEDANCE

REAL= 7.11911492

IMAG= 8.30592614

FEM1. Piston in Infinite Baffle with Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.90000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.13355	0.13613	0.13579	0.13968	0.14245	0.14650	0.15200	0.15661	0.16354
-48.071	-47.085	-47.932	-48.529	-49.115	-50.453	-51.005	-52.250	-53.654
0.15353	0.0	0.15462	0.0	0.15841	0.0	0.16390	0.0	0.16924
4.213	0.0	-1.440	0.0	-14.777	0.0	-32.507	0.0	-54.061
0.17814	0.17905	0.17866	0.17980	0.18045	0.18019	0.18079	0.17760	0.17702
56.454	54.050	45.619	33.786	20.005	2.857	-14.822	-34.395	-54.733
0.22101	0.0	0.21494	0.0	0.20761	0.0	0.19663	0.0	0.18179
109.867	0.0	91.552	0.0	50.888	0.0	0.578	0.0	-55.698
0.28014	0.27153	0.26876	0.25401	0.24446	0.22875	0.21540	0.19977	0.18712
162.646	153.347	136.330	109.466	80.474	47.889	14.864	-20.367	-56.128
0.35583	0.0	0.33268	0.0	0.28309	0.0	0.23312	0.0	0.19191
-142.184	0.0	175.666	0.0	104.587	0.0	25.969	0.0	-56.922
0.63930	0.54709	0.46567	0.38422	0.32637	0.28326	0.24769	0.21827	0.19454
-79.378	-111.828	-151.062	167.994	124.652	79.864	35.119	-10.397	-57.420
1.20887	0.0	0.59822	0.0	0.36241	0.0	0.25955	0.0	0.19818
-53.483	0.0	-131.342	0.0	136.976	0.0	40.911	0.0	-57.750
1.46254	0.97506	0.63836	0.48057	0.37286	0.31430	0.26065	0.22686	0.19239
-49.631	-85.219	-118.876	-168.541	143.663	93.398	43.164	-6.174	-58.558

ACOUSTIC IMPEDANCE

REAL= 11.96803122

IMAG= 7.24443414

FEM2. Piston in Infinite Baffle Without Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.10000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.00336 -104.805	0.00335 -104.666	0.00335 -104.877	0.00335 -104.952	0.00335 -105.151	0.00335 -105.318	0.00335 -105.517	0.00335 -105.741	0.00335 -106.040
0.00336 -98.047	0.0 0.0	0.00336 -98.930	0.0 0.0	0.00335 -100.924	0.0 0.0	0.00335 -103.435	0.0 0.0	0.00334 -106.234
0.00340 -88.884	0.00339 -90.120	0.00338 -91.277	0.00337 -93.588	0.00336 -96.099	0.00335 -98.663	0.00335 -101.366	0.00334 -103.939	0.00334 -106.480
0.00353 -77.901	0.0 0.0	0.00347 -82.123	0.0 0.0	0.00338 -90.469	0.0 0.0	0.00335 -99.231	0.0 0.0	0.00334 -106.708
0.00396 -62.364	0.00391 -63.401	0.00369 -70.759	0.00355 -77.149	0.00344 -84.376	0.00338 -91.204	0.00335 -97.300	0.00334 -102.433	0.00334 -106.878
0.00547 -40.418	0.0 0.0	0.00427 -55.516	0.0 0.0	0.00352 -78.541	0.0 0.0	0.00335 -95.428	0.0 0.0	0.00334 -107.024
0.00990 -21.154	0.00692 -31.017	0.00497 -45.494	0.00407 -59.634	0.00363 -73.085	0.00343 -84.648	0.00336 -93.930	0.00334 -101.277	0.00334 -107.089
0.01595 -13.011	0.0 0.0	0.00581 -37.842	0.0 0.0	0.00373 -69.199	0.0 0.0	0.00336 -92.929	0.0 0.0	0.00334 -107.128
0.01794 -11.564	0.00936 -22.554	0.00676 -31.885	0.00462 -49.896	0.00376 -68.341	0.00347 -82.022	0.00337 -92.074	0.00334 -100.682	0.00334 -107.263

ACOUSTIC IMPEDANCE

REAL= 0.45053255

IMAG= 1.68881084

FEM2. Piston in Infinite Baffle Without Spherical Correction
Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.30000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.01798	0.01793	0.01796	0.01793	0.01792	0.01790	0.01788	0.01785	0.01784
-144.904	-144.753	-145.027	-145.273	-145.624	-145.937	-146.307	-146.713	-147.337
0.01810	0.0	0.01805	0.0	0.01795	0.0	0.01786	0.0	0.01780
-125.114	0.0	-127.452	0.0	-132.844	0.0	-139.799	0.0	-147.655
0.01959	0.01949	0.01909	0.01871	0.01835	0.01806	0.01791	0.01781	0.01779
-100.100	-102.660	-105.856	-111.665	-118.391	-125.608	-133.295	-140.764	-148.153
0.02430	0.0	0.02236	0.0	0.01945	0.0	0.01800	0.0	0.01777
-75.714	0.0	-83.713	0.0	-102.782	0.0	-126.700	0.0	-148.599
0.03414	0.03293	0.02866	0.02485	0.02153	0.01932	0.01821	0.01781	0.01775
-53.351	-54.844	-63.818	-73.985	-88.003	-104.117	-120.662	-135.724	-148.960
0.05583	0.0	0.03971	0.0	0.02441	0.0	0.01847	0.0	0.01774
-33.246	0.0	-46.246	0.0	-75.995	0.0	-115.134	0.0	-149.232
0.10358	0.07417	0.05134	0.03723	0.02759	0.02167	0.01876	0.01784	0.01772
-18.598	-25.978	-37.069	-50.025	-66.867	-87.835	-110.709	-131.895	-149.337
0.16325	0.0	0.06278	0.0	0.03018	0.0	0.01902	0.0	0.01771
-12.217	0.0	-31.053	0.0	-61.282	0.0	-107.811	0.0	-149.440
0.18311	0.10276	0.07324	0.04636	0.03115	0.02309	0.01908	0.01784	0.01773
-11.038	-19.634	-26.699	-40.885	-59.783	-81.850	-105.829	-130.124	-149.601

ACOUSTIC IMPEDANCE

REAL= 1.42595320

IMAG= 5.82615666

FEM2. Piston in Infinite Baffle Without Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.60000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.07266 75.250	0.07322 75.183	0.07325 75.287	0.07406 74.767	0.07471 74.563	0.07550 74.173	0.07662 73.887	0.07751 73.490	0.07886 72.755
0.07526 113.624	0.0 0.0	0.07537 109.638	0.0 0.0	0.07604 100.072	0.0 0.0	0.07758 87.300	0.0 0.0	0.07984 72.632
0.09072 155.462	0.09118 153.276	0.08809 147.519	0.08569 138.936	0.08312 128.045	0.08073 114.949	0.08012 100.845	0.07999 86.413	0.08122 72.166
0.12570 -169.876	0.0 0.0	0.11660 179.760	0.0 0.0	0.09770 153.364	0.0 0.0	0.08352 113.852	0.0 0.0	0.08208 71.725
0.16452 -140.455	0.15811 -144.033	0.15269 -154.753	0.13677 -168.533	0.11941 173.808	0.10150 151.879	0.08895 125.679	0.08323 97.443	0.08298 71.349
0.20198 -105.068	0.0 0.0	0.18397 -131.383	0.0 0.0	0.14291 -171.060	0.0 0.0	0.09487 135.226	0.0 0.0	0.08370 71.063
0.31905 -60.357	0.25874 -85.177	0.22424 -113.375	0.19598 -137.009	0.16406 -160.116	0.12955 174.279	0.10063 142.675	0.08637 105.627	0.08398 70.978
0.53874 -38.377	0.0 0.0	0.25787 -99.898	0.0 0.0	0.17903 -153.381	0.0 0.0	0.10522 147.318	0.0 0.0	0.08446 70.782
0.62510 -34.649	0.37118 -64.484	0.27339 -89.206	0.22279 -122.932	0.18621 -150.469	0.14495 -177.612	0.10611 149.403	0.08789 108.951	0.08400 70.736

ACOUSTIC IMPEDANCE

REAL= 6.60598018

IMAG= 6.99861861

FEM2. Piston in Infinite Baffle Without Spherical Correction Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.90000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.14431 -53.197	0.14672 -52.175	0.14637 -53.027	0.15017 -53.632	0.15273 -54.224	0.15675 -55.581	0.16209 -56.141	0.16658 -57.417	0.17368 -58.849
0.15527 4.211	0.0 0.0	0.15551 -2.094	0.0 0.0	0.15868 -17.075	0.0 0.0	0.16676 -36.728	0.0 0.0	0.17922 -59.282
0.19974 58.374	0.19974 56.119	0.19571 48.170	0.19144 36.540	0.18594 22.225	0.18015 3.375	0.17929 -16.747	0.17998 -38.681	0.18731 -59.978
0.24969 105.800	0.0 0.0	0.24654 89.399	0.0 0.0	0.22950 53.048	0.0 0.0	0.19575 0.933	0.0 0.0	0.19235 -60.977
0.28526 159.428	0.27934 149.237	0.28405 131.313	0.28240 105.178	0.27807 79.220	0.25105 49.966	0.21948 16.786	0.19803 -22.890	0.19796 -61.400
0.38208 -140.381	0.0 0.0	0.33235 174.968	0.0 0.0	0.31326 100.366	0.0 0.0	0.24385 28.377	0.0 0.0	0.20267 -62.090
0.71787 -83.015	0.62007 -111.521	0.49012 -148.131	0.37796 166.954	0.34305 119.408	0.31944 78.005	0.26506 37.436	0.21600 -11.265	0.20493 -62.402
1.29487 -58.602	0.0 0.0	0.66050 -128.609	0.0 0.0	0.36737 132.192	0.0 0.0	0.28154 42.900	0.0 0.0	0.20818 -62.657
1.55387 -54.747	1.10882 -88.011	0.72188 -117.150	0.48296 -165.799	0.37122 139.523	0.34985 89.706	0.28443 45.091	0.22566 -6.520	0.20580 -63.560

ACOUSTIC IMPEDANCE

REAL= 13.60456934

IMAG= 6.68581093

NUMBER OF INCREMENTS OF OMEGA=12

FEM3. Diaphragm in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.10000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.00126 -46.066	0.00127 -46.009	0.00126 -46.082	0.00126 -46.125	0.00126 -46.201	0.00126 -46.253	0.00126 -46.325	0.00126 -46.401	0.00125 -46.553
0.00145 -40.277	0.0 0.0	0.00142 -40.861	0.0 0.0	0.00138 -42.263	0.0 0.0	0.00132 -44.206	0.0 0.0	0.00125 -46.607
0.00169 -34.393	0.00167 -34.906	0.00163 -35.646	0.00158 -36.926	0.00151 -38.500	0.00145 -40.263	0.00138 -42.288	0.00132 -44.416	0.00125 -46.729
0.00202 -28.783	0.0 0.0	0.00190 -30.623	0.0 0.0	0.00167 -34.859	0.0 0.0	0.00144 -40.492	0.0 0.0	0.00125 -46.834
0.00255 -22.849	0.00247 -23.542	0.00227 -25.719	0.00206 -28.296	0.00185 -31.592	0.00166 -35.131	0.00150 -38.961	0.00137 -42.883	0.00125 -46.928
0.00351 -16.563	0.0 0.0	0.00281 -20.740	0.0 0.0	0.00204 -28.695	0.0 0.0	0.00156 -37.633	0.0 0.0	0.00126 -47.002
0.00563 -10.325	0.00454 -12.815	0.00340 -17.136	0.00271 -21.541	0.00221 -26.433	0.00186 -31.460	0.00161 -36.599	0.00141 -41.822	0.00126 -47.042
0.01060 -5.490	0.0 0.0	0.00407 -14.325	0.0 0.0	0.00235 -24.849	0.0 0.0	0.00164 -35.925	0.0 0.0	0.00127 -47.070
0.01427 -4.084	0.00689 -8.472	0.00461 -12.642	0.00317 -18.450	0.00242 -24.176	0.00196 -29.994	0.00166 -35.525	0.00142 -41.336	0.00125 -47.116

ACOUSTIC IMPEDANCE

REAL= 0.06370248

IMAG= 0.63658158

FEM3. Diaphragm in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.30000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.01098 -136.759	0.01099 -136.686	0.01098 -136.789	0.01098 -136.962	0.01098 -137.141	0.01098 -137.291	0.01098 -137.484	0.01099 -137.691	0.01100 -138.120
0.01254 -119.497	0.0 0.0	0.01238 -121.205	0.0 0.0	0.01201 -125.339	0.0 0.0	0.01153 -131.107	0.0 0.0	0.01101 -138.254
0.01463 -101.915	0.01451 -103.289	0.01418 -105.574	0.01374 -109.335	0.01323 -113.990	0.01267 -119.283	0.01211 -125.309	0.01155 -131.681	0.01102 -138.590
0.01762 -84.934	0.0 0.0	0.01660 -90.420	0.0 0.0	0.01463 -103.097	0.0 0.0	0.01267 -119.916	0.0 0.0	0.01103 -138.881
0.02220 -67.248	0.02150 -69.403	0.01987 -75.697	0.01805 -83.460	0.01624 -93.262	0.01462 -103.847	0.01323 -115.263	0.01204 -127.029	0.01103 -139.149
0.03050 -48.708	0.0 0.0	0.02459 -61.047	0.0 0.0	0.01796 -84.587	0.0 0.0	0.01374 -111.289	0.0 0.0	0.01108 -139.369
0.04919 -30.231	0.03997 -37.565	0.03023 -50.239	0.02404 -63.295	0.01961 -77.811	0.01648 -92.797	0.01417 -108.173	0.01244 -123.813	0.01113 -139.491
0.09381 -16.037	0.0 0.0	0.03635 -42.030	0.0 0.0	0.02092 -73.145	0.0 0.0	0.01447 -106.142	0.0 0.0	0.01118 -139.605
0.12726 -11.928	0.06189 -24.759	0.04096 -37.151	0.02817 -54.245	0.02152 -71.068	0.01740 -88.328	0.01461 -105.049	0.01257 -122.397	0.01100 -139.656

ACOUSTIC IMPEDANCE

REAL= 0.54011493

IMAG= 1.80323806

FEM3. Diaphragm in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN= 0.60000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.04011 86.160	0.04037 86.264	0.04032 86.271	0.04061 85.924	0.04081 85.762	0.04104 85.464	0.04143 85.251	0.04170 84.916	0.04217 84.235
0.04589 120.534	0.0 0.0	0.04555 117.261	0.0 0.0	0.04478 109.265	0.0 0.0	0.04374 98.077	0.0 0.0	0.04252 84.154
0.05313 155.411	0.05328 153.344	0.05217 148.517	0.05113 141.252	0.04982 132.317	0.04806 121.693	0.04654 109.960	0.04460 97.299	0.04302 83.725
0.06475 -169.810	0.0 0.0	0.06160 179.198	0.0 0.0	0.05551 153.895	0.0 0.0	0.04907 120.789	0.0 0.0	0.04330 83.276
0.08137 -134.539	0.07902 -139.386	0.07479 -151.060	0.06843 -167.057	0.06284 173.856	0.05688 152.791	0.05193 130.427	0.04735 106.978	0.04360 82.908
0.11005 -98.250	0.0 0.0	0.09217 -122.886	0.0 0.0	0.07041 -169.083	0.0 0.0	0.05446 138.362	0.0 0.0	0.04398 82.499
0.17825 -60.629	0.14928 -75.537	0.11776 -100.607	0.09472 -126.915	0.07840 -155.495	0.06604 175.010	0.05658 144.687	0.04954 113.692	0.04418 82.261
0.35379 -32.026	0.0 0.0	0.14360 -84.780	0.0 0.0	0.08441 -146.616	0.0 0.0	0.05820 148.773	0.0 0.0	0.04453 81.997
0.49338 -23.844	0.24710 -49.582	0.15898 -75.523	0.11251 -109.603	0.08665 -142.253	0.07088 -175.848	0.05841 150.585	0.05035 116.504	0.04358 81.877

ACOUSTIC IMPEDANCE

REAL= 1.92789152

IMAG= 2.94996724

FEM3. Diaphragm in Infinite Baffle with Spherical Contribution Added at the Non-Reflection Boundary

VALUE OF OMEGA FOR THIS RUN=   0.90000

ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE

(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)

0.07730	0.07845	0.07809	0.07959	0.08049	0.08183	0.08384	0.08522	0.08779
-49.379	-48.551	-49.129	-49.629	-49.967	-50.990	-51.274	-52.158	-53.220
0.08865	0.0	0.08853	0.0	0.08873	0.0	0.08923	0.0	0.08967
2.627	0.0	-2.737	0.0	-15.424	0.0	-32.433	0.0	-53.302
0.10270	0.10326	0.10171	0.10117	0.09995	0.09814	0.09701	0.09404	0.09265
54.794	52.062	44.285	32.852	19.545	2.927	-14.369	-33.615	-53.661
0.12664	0.0	0.12139	0.0	0.11318	0.0	0.10389	0.0	0.09421
106.954	0.0	89.798	0.0	50.900	0.0	1.436	0.0	-54.370
0.15738	0.15560	0.14845	0.13865	0.13097	0.12056	0.11226	0.10323	0.09610
159.354	151.303	134.710	109.309	81.395	49.091	16.248	-18.930	-54.548
0.20929	0.0	0.18441	0.0	0.14913	0.0	0.11988	0.0	0.09793
-147.999	0.0	175.456	0.0	106.088	0.0	27.836	0.0	-55.160
0.33963	0.29929	0.24879	0.20130	0.16988	0.14553	0.12613	0.11086	0.09875
-89.983	-112.628	-149.068	170.451	127.379	82.497	37.463	-8.422	-55.516
0.72350	0.0	0.31667	0.0	0.18668	0.0	0.13143	0.0	0.10030
-47.294	0.0	-126.271	0.0	140.439	0.0	43.543	0.0	-55.774
1.05780	0.55984	0.34658	0.25245	0.18885	0.15994	0.13127	0.11438	0.09734
-35.353	-73.512	-113.561	-163.877	147.129	96.461	45.983	-3.977	-56.523

ACOUSTIC IMPEDANCE

REAL=   3.57037792

IMAG=   3.14224531

APPENDIX B - COMPUTER PROGRAM

```
C       THE FINITE ELEMENT STUDY OF ACOUSTIC WAVES CREATED BY
C       A MECHANISM ACTING ON A FLUID SUBSTANCE AT THE CENTER
C       OF AN INFINITE WALL.  THE MECHANISM MAY BE A PISTON,
C       DIAPHRAM, OR SOME OTHER MECHANICAL MEANS OF APPLYING
C       A FORCE TO THE FLUID.  THE FLUID REGION AWAY FROM THE
C       WALL IS CONSIDERED TO BE INFINITE.
C
C
C       THIS PROGRAM IS DESIGNED TO TAKE ONE 8 NODED ELEMENT
C       AND SUBDIVIDE IT INTO ANY NUMBER OF SMALLER ELEMENTS
C       BY SPECIFYING THE NUMBER OF DIVISIONS DESIRED IN BOTH
C       THE X AND Y DIRECTION.  EACH SUBDIVIDED ELEMENT WILL
C       AUTOMATICALLY BE ASSIGNED 9 GLOBAL NODE NUMBERS, 8 OF
C       WHICH ARE ON THE BOUNDARY AND THE 9TH IS LOCATED AT
C       THE CENTER.  THE 9TH (CENTER) NODE IS USED TO PROVIDE
C       A MEANS FOR FORMULATING LEGIBLE OUTPUT.  COORDINATES
C       OF ALL GLOBAL NODES ARE THEN CALCULATED AND PLOTTED
C       TO FORM THE FINITE ELEMENT MESH FOR THE PROBLEM TO BE
C       SOLVED.  NON-REFLECTION BOUNDARY NODE NUMBERS AND THE
C       NODE NUMBERS IN CONTACT WITH THE FORCING MECHANISM ARE
C       AUTOMATICALLY ASSIGNED IN THE PROGRAM.
C
C
C       -----DATA CARDS FOR SUBROUTINE INPUT
C       ..............................................................
C
C          (1) TITLE OF GRAPH OF THE FINITE ELEMENT MESH WHICH
C              WILL BE GENERATED AND PLOTTED IN THE PROGRAM.
C              (TWO CARDS)  FORMAT(6A8) TITLE OF GRAPH
C
C          (2) NUMBER OF FLUID ELEMENTS,NUMBER OF NODES PER
C              ELEMENT AND TOTAL NUMBER OF SYSTEM NODES.
C              EACH ELEMENT HAS 8 NODES ON THE BOUNDARY AND
C              ONE MIDDLE NODE.  THE MIDDLE NODE IS USED ONLY
C              TO ASSIST IN PRINTING OUT THE RESULTS IN ARRAY FORM.
C              (ONE CARD)  FORMAT(3I10)NE,MT,NNODT
C
C          (3) X AND Y COORDINATES OF THE EIGHT MAIN NODES
C              ASSIGNED TO THE BOUNDARY OF THE REGION TO BE
C              STUDIED.  (ONE CARD PER NODE)
C              FORMAT(I2,2F10.5) K,XM(K),YM(K)
C
C          (4) NUMBER OF SUBDIVISIONS REQUIRED OF THE TOTAL
C              REGION IN BOTH THE X AND Y DIRECTION.  (ONE CARD)
C              FORMAT(2I5) NXDIV,NYDIV
C
C          (5) TOTAL NUMBER OF NODES LOCATED ON THE NON-REFLECTION
C              BOUNDARY INCLUDING TWO VALUES OF XI DEFINING THE
```

```
C         RANGE OF THE NON-REFLECTION BOUNDARY ALONG THE
C         FACE ETA=1 AND THE TWO VALUES OF ETA WHICH DEFINE
C         THE RANGE OF THE NON-REFLECTION BOUNDARY ALONG
C         THE FACE XI=1.    (ONE CARD)
C         FORMAT(I2,8X,4F10.5)NRBN,XI1,XI2,ETA1,ETA2
C
C     (6) SPEED OF SOUND AND THE FLUID DENSITY.
C         (ONE CARD)   FORMAT(2F10.5)SOUND,FLDEN
C
C     (7) NUMBER OF NODES IN WHICH THE FORCING MECHANISM
C         IS IN CONTACT AND TWO VALUES OF ETA DEFINING THE
C         RANGE OF THE FORCING MECHANISM ACTING ON THE
C         FACE XI=-1.  (ONE CARD)
C         FORMAT(I2,8X,2F10.5)NP,ETA1,ETA2
C
C     (8) AMPLITUDE OF THE FORCING FUNCTION AT THE NODES
C         WITH WHICH THE FORCING MECHANISM IS IN CONTACT.
C         AMPLITUDES READ FROM THE TOP DOWNWARD.
C         (ONE CARD)   FORMAT(9F8.4)(AMP(I),I=1,NP)
C
C
C     (9) INITIAL VALUE OF OMEGA FOR WHICH PROBLEM IS TO
C         BE SOLVED, INCREMENT OF OMEGA FOR SUCCEEDING
C         SOLUTIONS, AND TOTAL NUMBER OF SOLUTIONS REQUIRED.
C         (ONE CARD)   FORMAT(2F10.5,I2) OMEGA,OINCR,NO
C   ..............................................................
C
      CALL INPUT
      CALL HQFORM
      CALL FDAMP
      CALL FORCE
      CALL PROB
      STOP
      END


      SUBROUTINE INPUT
C
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81),DB(81)
      COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
      COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XXD(8),YYD(8),AJ(2,2),BL(3,3)
      COMMON SOUND,FLDEN,OMEGA,OINCR
      COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC
      COMMON /TITLE/ITITLE(12)
```

```
C
C
C     TITLE OF GRAPH ON WHICH CORNER NODE POINTS WILL BE PLOTTED
C
      READ(5,142)(ITITLE(I),I=1,6)
      READ(5,142)(ITITLE(I),I=7,12)
  142 FORMAT(6A8)
      WRITE(6,143)
  143 FORMAT('1','TITLE OF GRAPH OF CORNER NODE POINTS TO BE PLOTTED'/)
      WRITE(6,144) ITITLE
  144 FORMAT(4X,6A8/)
C
C     NUMBER OF FLUID ELEMENTS, NODES PER ELEMENT,TOTAL
C     NODES, AND NUMBER OF WORKING NODES.
C
      READ(5,100) NE,MT,NNODT
  100 FORMAT(3I10)
      WRITE(6,101) NE
  101 FORMAT('0','NUMBER OF ELEMENTS=',I2)
      WRITE(6,103)MT
  103 FORMAT('0','NUMBER OF NODES PER ELEMENT INCLUDING CENTER NODES=',
     1I2)
      WRITE(6,105) NNODT
  105 FORMAT('0','NUMBER OF TOTAL SYSTEM NODES INCLUDING CENTER NODES='
     1,I2)
      M=MT-1
      WRITE(6,106) M
  106 FORMAT('0','NUMBER OF NODES PER ELEMENT EXCLUDING CENTER NODES=',
     1I2)
      NNOD=NNODT-NE
      WRITE(6,107)NNOD
  107 FORMAT('0','NUMBER OF SYSTEM NODES EXCLUDING CENTER NODES=',I2)
C
C     X AND Y COORDINATES FOR MAIN EIGHT NODES
C
      DO 13 I=1,NNODT
      X(I)=0.D0
   13 Y(I)=0.D0
      WRITE(6,120)
  120 FORMAT('0','MAIN NODE NO',6X,'X COORD',6X,'Y COORD')
      DO 12 I=1,M
      READ(5,121) K,XM(K),YM(K)
  121 FORMAT(I2,2F10.5)
   12 WRITE(6,122) K,XM(K),YM(K)
  122 FORMAT(5X,I2,11X,F10.5,3X,F10.5)
C
C     ARRAY OF NODE NUMBERS AND FORMATION OF X AND Y COORDINATES.
C
```

```
      READ(5,170)NXDIV,NYDIV
  170 FORMAT(2I5)
      WRITE(6,171) NXDIV
  171 FORMAT('0','NUMBER OF ELEMENT DIVISIONS IN X DIRECTION=',I3)
      WRITE(6,172) NYDIV
  172 FORMAT('0','NUMBER OF ELEMENT DIVISIONS IN THE Y DIRECTION=',I3)
C
      CALL NODE(NXDIV,NYDIV)
      CALL XYFORM(NXDIV,NYDIV)
C
C     NUMBER OF NODES LOCATED ON THE NONREFLECTION BOUNDARY
C     AND THEIR RESPECTIVE NODE NUMBERS
C
      READ(5,114) NRBN,XI1,XI2,ETA1,ETA2
  114 FORMAT(I2,8X,4F10.5)
      WRITE(6,131)NRBN
  131 FORMAT('0','NUMBER OF NODES ON NONREFLECTION BOUNDARY=',I2)
      WRITE(6,115) XI1,XI2
  115 FORMAT('0','NONREFLECTION BOUNDARY ALONG TOP OF ELEMENT IS FROM
     1XI=',F5.2,1X,'TO XI=',F5.2)
      WRITE(6,116) ETA1,ETA2
  116 FORMAT('0','NONREFLECTION BOUNDARY ALONG RIGHT SIDE OF ELEMENT IS
     1FROM ETA=',F5.2,1X,'TO ETA=',F5.2)
      XINCR=1.D0/NXDIV
      YINCR=1.D0/NYDIV
      NA=2*NXDIV*NYDIV+NXDIV+NYDIV+1
      NB=NYDIV*(2*NXDIV+1)
      NC=NXDIV
      DO 10 I=1,NRBN
      IF(XI1.EQ.XI2) GO TO 118
      EN=NA-NB+NC*XI1
      XI1=XI1+XINCR
      IF(XI1.LT.XI2.OR.XI1.EQ.XI2) GO TO 20
  118 EN=NA-NB*ETA1+NC
      ETA1=ETA1-YINCR
   20 II=EN
      NDAMP(I)=NOW(II)
   10 CONTINUE
      WRITE(6,132)
  132 FORMAT('0','NODE NUMBERS LOCATED ON THE NONREFLECTION BOUNDARY')
      WRITE(6,133)(NDAMP(I),I=1,NRBN)
  133 FORMAT(5X,I3/)
C
C     SPEED OF SOUND AND FLUID DENSITY
C
      READ(5,102)SOUND,FLDEN
  102 FORMAT(2F10.5)
      WRITE(6,126) SOUND
```

```
  126 FORMAT('0','SPEED OF SOUND =',F10.5)
      WRITE(6,141)FLDEN
  141 FORMAT('0','DENSITY OF FLUID=',F10.5)
C
C     NUMBER OF FLUID NODES AND GLOBAL NODES NUMBERS WHICH
C     FORCING MECHANISM IS IN CONTACT.
C
      READ(5,148)NP,ETA1,ETA2
  148 FORMAT(I2,8X,2F10.5)
      WRITE(6,149)NP
  149 FORMAT('0','NUMBER OF NODES IN CONTACT WITH PISTON=',I4)
      WRITE(6,152) ETA1,ETA2
  152 FORMAT('0','RANGE ON LEFT BOUNDARY WHICH PISTON IS IN CONTACT IS F
     1ROM ETA=',F5.2,1X,'TO ETA=',F5.2)
      WRITE(6,151)
  151 FORMAT('0','NODE NUMBERS IN CONTACT  WITH FORCING FUNCTION')
      DO 30 I=1,NP
      EN=NA-NB*ETA1-NC
      ETA1=ETA1-YINCR
      II=EN
      NPN(I)=NOW(II)
   30 WRITE(6,153)NPN(I)
  153 FORMAT(5X,I3)
C
C     AMPLITUDE OF THE FORCING FUNCTION AT THE NODE POINTS
C     WITH WHICH THE FORCING MECHANISM IS IN CONTACT.
C
      WRITE(6,138)
  138 FORMAT('0','AMPLITUDE OF PISTON AT THE INTERFACE NODES')
      WRITE(6,139)
  139 FORMAT('0',4X,'NODE',5X,'AMPLITUDE')
      READ(5,134)(AMP(I),I=1,NP)
  134 FORMAT(9F8.4)
      WRITE(6,137)(NPN(I),AMP(I),I=1,NP)
  137 FORMAT(6X,I2,2X,F16.8/)
C
C     INITIAL VALUE OF OMEGA IN THE PISTON FORCING FUNCTION, INCREMENT VALUE
C     OF OMEGA FOR SUCCEEDING SOLUTIONS, AND TOTAL NUMBER OF OMEGAS FOR
C     WHICH PROBLEM IS TO BE CALCULATED
C
      READ(5,160) OMEGA,OINCR,NO
  160 FORMAT(2F10.5,I2)
      WRITE(6,161) OMEGA
  161 FORMAT('0','INITIAL VALUE OF OMEGA=',F10.5)
      WRITE(6,162) OINCR
  162 FORMAT('0','INCREMENT VALUE OF OMEGA=',F10.5)
      WRITE(6,163) NO
  163 FORMAT('0','NUMBER OF INCREMENTS OF OMEGA=',I2)
```

```
      RETURN
      END

      SUBROUTINE HQFORM
C
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81),DB(81)
      COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
      COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XXD(8),YYD(8),AJ(2,2),BL(3,3)
      COMMON SOUND,FLDEN,OMEGA,OINCR
      COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC
C
C     SETTING ALL ELEMENTS OF Q AND H EQUAL TO ZERO
C
      DO 20 I=1,NNOD
      DO 20 J=1,NNOD
      Q(I,J)=0.D0
   20 H(I,J)=0.D0
C
C     GAUSS QUADRATURE ABSCISSA VALUES AND WEIGHT COEFFICIENTS
C
      GA(1)= .861136311594053D0
      GA(2)= .339981043584856D0
      GA(3)=-.339981043584856D0
      GA(4)=-.861136311594053D0
      GH(1)= .347854845137454D0
      GH(2)= .652145154862546D0
      GH(3)=-.652145154862546D0
      GH(4)= .347854845137454D0
C
C     FORMULATION OF THE ELEMENT HE AND QE MATRICES USING THE 4 TERM GAUSS
C     QUADRATURE NUMERICAL INTEGRATION SCHEME
C
      DO 21 L=1,NE
      DO 22 I=1,M
      DO 22 J=1,M
      QE(I,J)=0.D0
   22 HE(I,J)=0.D0
C
      DO 23 LA=1,4
      DO 23 LB=1,4
      GX=GA(LA)
      GY=GA(LB)
      SF(1)= .25D0*(1.D0-GX)*(1.D0-GY)*(-GX-GY-1.D0)
```

```
      SF(2)= .50D0*(1.D0-GX**2)*(1.D0-GY)
      SF(3)= .25D0*(1.D0+GX)*(1.D0-GY)*(+GX-GY-1.D0)
      SF(4)= .50D0*(1.D0+GX)*(1.D0-GY**2)
      SF(5)= .25D0*(1.D0+GX)*(1.D0+GY)*(+GX+GY-1.D0)
      SF(6)= .50D0*(1.D0-GX**2)*(1.D0+GY)
      SF(7)= .25D0*(1.D0-GX)*(1.D0+GY)*(-GX+GY-1.D0)
      SF(8)= .50D0*(1.D0-GX)*(1.D0-GY**2)
      XD(1)= .25D0*(1.D0-GY)*(2.D0*GX+GY)
      XD(2)= -1.D0*(1.D0-GY)*(GX)
      XD(3)= .25D0*(1.D0-GY)*(2.D0*GX-GY)
      XD(4)= .50D0*(1.D0-GY**2)
      XD(5)= .25D0*(1.D0+GY)*(2.D0*GX+GY)
      XD(6)= -1.D0*(1.D0+GY)*(GX)
      XD(7)= .25D0*(1.D0+GY)*(2.D0*GX-GY)
      XD(8)= -.50D0*(1.D0-GY**2)
      YD(1)= .25D0*(1.D0-GX)*(2.D0*GY+GX)
      YD(2)= -.50D0*(1.D0-GX**2)
      YD(3)= .25D0*(1.D0+GX)*(2.D0*GY-GX)
      YD(4)= -1.D0*(1.D0+GX)*(GY)
      YD(5)= .25D0*(1.D0+GX)*(2.D0*GY+GX)
      YD(6)= .50D0*(1.D0-GX**2)
      YD(7)= .25D0*(1.D0-GX)*(2.D0*GY-GX)
      YD(8)= -1.D0*(1.D0-GX)*(GY)
      DO 24 I=1,2
      DO 24 J=1,2
   24 AJ(I,J)=0.D0
      DO 26 I=1,M
      AJ(1,1)=AJ(1,1) + XD(I) * X(NN(L,I))
      AJ(1,2)=AJ(1,2) + XD(I) * Y(NN(L,I))
      AJ(2,1)=AJ(2,1) + YD(I) * X(NN(L,I))
   26 AJ(2,2)=AJ(2,2) + YD(I) * Y(NN(L,I))
      ADET = AJ(1,1) * AJ(2,2) -AJ(1,2) * AJ(2,1)
      TEMP = AJ(1,1)
      AJ(1,1) = AJ(2,2)/ADET
      AJ(1,2) = -AJ(1,2)/ADET
      AJ(2,1) = -AJ(2,1)/ADET
      AJ(2,2) = TEMP/ADET
      DO 28 I=1,M
      XXD(I) = AJ(1,1) * XD(I) + AJ(1,2) * YD(I)
   28 YYD(I) = AJ(2,1) * XD(I) + AJ(2,2) * YD(I)
      YY=0.D0
      DO 29 I=1,M
   29 YY=YY+SF(I)*Y(NN(L,I))
      COEFH=ADET*GH(LA) * GH(LB) * YY
      COEFQ=COEFH/SOUND**2
      DO 23 LC=1,M
      DO 23 LD=1,M
      FUNH=COEFH*(XXD(LC)*XXD(LD)+YYD(LC)*YYD(LD))
```

```
      FUNQ=COEFQ*SF(LC)*SF(LD)
      QE(LC,LD) = QE(LC,LD) + FUNQ
   23 HE(LC,LD) = HE(LC,LD) + FUNH
C
C     ADDING ELEMENT HE AND QE MATRICFS TO TOTAL FLUID MATRICES
C
   25 DO 21 I=1,M
      DO 21 J=1,M
      II=NN(L,I)
      JJ=NN(L,J)
      Q(II,JJ)=Q(II,JJ)+QE(I,J)
   21 H(II,JJ)=H(II,JJ)+HE(I,J)
      RETURN
      END


      SUBROUTINE FDAMP
C
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81),DB(81)
      COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON RD(50),RDS(50),RDSS(50),XM(3),YM(8),AMP(25)
      COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XXD(8),YYD(8),AJ(2,2),BL(3,3)
      COMMON SOUND,FLDEN,OMEGA,OINCR
      COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC
C
C     SETTING ELEMENTS OF DAMPING VECTOR EQUAL TO ZERO
      DO 40 I=1,NRBN
      RD(I)=0.D0
      RDSS(I)=0.D0
   40 RDS(I)=0.D0
C
C     FORMULATION OF THE VECTORS RD,RDS,AND RDSS BY THE 4 POINT GAUSS
C     QUADRATURE NUMERICAL INTEGRATION SCHEME.  VECTOR RD REPRESENTS
C     ELEMENTS ON THE MAIN DIAGONAL, VECTOR RDS REPRESENTS THE FIRST OFF
C     DIAGONAL ELEMENTS, AND VECTOR RDSS REPRESENTS THE SECOND OFF
C     DIAGONAL ELEMENTS
C
      DO 42 J=3,NRBN,2
      K=J-1
      I=J-2
      II=NDAMP(I)
      JJ=NDAMP(J)
      KK=NDAMP(K)
      DO 42 L=1,4
      GX=GA(L)
```

```
      SF(1)= .50D0*(1.D0-GX)*(-GX)
      SF(2)= 1.D0*(1.D0-GX**2)
      SF(3)= .50D0*(1.D0+GX)*(GX)
      XD(1)=-.50D0+GX
      XD(2)=-2.D0*GX
      XD(3)=+.50D0+GX
      XDIF=XD(1)*X(II)+XD(2)*X(KK)+XD(3)*X(JJ)
      YDIF=XD(1)*Y(II)+XD(2)*Y(KK)+XD(3)*Y(JJ)
      SDIF=DSQRT(XDIF*XDIF + YDIF*YDIF)
      YY=SF(1)*Y(II)+SF(2)*Y(KK)+SF(3)*Y(JJ)
      COEF=YY*SDIF*GH(L)/SOUND
      RD(I)=RD(I)+COEF*SF(1)*SF(1)
      RD(K)=RD(K)+COEF*SF(2)*SF(2)
      RD(J)=RD(J)+COEF*SF(3)*SF(3)
      RDS(I)=RDS(I)+COEF*SF(1)*SF(2)
      RDS(K)=RDS(K)+COEF*SF(2)*SF(3)
   42 RDSS(I)=RDSS(I)+COEF*SF(1)*SF(3)
C
C     ADDING SPHERICAL CONTRIBUTION TO THE MATRIX H
C
      R=Y(1)
      DO 44 I=1,NRBN
      II=NDAMP(I)
      S=RD(I)*SOUND/R
   44 H(II,II)=H(II,II)+S
      DO 46 I=2,NRBN
      J=I-1
      JJ=NDAMP(J)
      II=NDAMP(I)
      S=RDS(I)*SOUND/R
      H(JJ,II)=H(JJ,II)+S
   46 H(II,JJ)=H(II,JJ)+S
      DO 48 I=3,NRBN,2
      J=I-2
      JJ=NDAMP(J)
      II=NDAMP(I)
      S=RDSS(I)*SOUND/R
      H(JJ,II)=H(JJ,II)+S
   48 H(II,JJ)=H(II,JJ)+S
      RETURN
      END

      SUBROUTINE FORCE
C
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81),DB(81)
```

```
      COMMON  H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON  RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
      COMMON  GA(4),GH(4),SF(8),XD(8),YD(8),XXD(8),YYD(8),AJ(2,2),BL(3,3)
      COMMON  SOUND,FLDEN,OMEGA,OINCR
      COMMON  NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON  NE,M,NNOD,NRBN,NP,NO,NNODT,NC
C
C     SETTING ALL ELEMENTS OF THE PISTON FORCING FUNCTION EQUAL TO ZERO
C
      DO 50  I=1,NNOD
   50 B(I)=0.DO
C
C     FORMULATION OF THE FORCING FUNCTION USING THE 4 POINT
C     GAUSS QUADRATURE NUMERICAL INTEGRATION SCHEME
C
      DO 52  J=3,NP,2
      DO 51  LC=1,3
      DO 51  LD=1,3
   51 BL(LC,LD)=0.DO
      K=J-1
      I=J-2
      II=NPN(I)
      KK=NPN(K)
      JJ=NPN(J)
      ED=DSQRT((Y(II)-Y(JJ))**2 + (X(II)-X(JJ))**2)
      DO 54  L=1,4
      GX=GA(L)
      SF(1)=  .50DO*(1.DO+GX)*(GX)
      SF(2)=  1.DO*(1.DO-GX**2)
      SF(3)=  .50DO*(1.DO-GX)*(-GX)
      YY=SF(1)*Y(II)+SF(2)*Y(KK)+SF(3)*Y(JJ)
      COEF=(ED/2.DO)*YY*GH(L)
      DO 54  LA=1,3
      DO 54  LB=1,3
   54 BL(LA,LB)=BL(LA,LB)+COEF*SF(LA)*SF(LB)
      B(II)=B(II)+BL(1,1)*AMP(I)+BL(1,2)*AMP(K)+BL(1,3)*AMP(J)
      B(KK)=B(KK)+BL(2,1)*AMP(I)+BL(2,2)*AMP(K)+BL(2,3)*AMP(J)
   52 B(JJ)=B(JJ)+BL(3,1)*AMP(I)+BL(3,2)*AMP(K)+BL(3,3)*AMP(J)
      DO 56  I=1,NP
      II=NPN(I)
   56 B(II)=B(II)*FLDEN
      RETURN
      END
```

```
      SUBROUTINE PROB
C
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81),DB(81)
      COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
      COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XXD(8),YYD(8),AJ(2,2),BL(3,3)
      COMMON SOUND,FLDEN,OMEGA,OINCR
      COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC
      DIMENSION ABPRES(100),PHASE(100)
C
C     INTRODUCING INITIAL VALUE OF OMEGA AND SUBSEQUENT VALUES
C
      DO 70 KOUNT=1,NO
      WRITE(6,600) OMEGA
  600 FORMAT('1','VALUE OF OMEGA FOR THIS RUN=',F10.5)
C
C     SETTING VALUES OF DSUM EQUAL TO ZERO
C
      DO 60 I=1,NNOD
      DO 60 J=1,NNOD
   60 DSUM(I,J)=(0.D0,0.D0)
C
C     SUMMING ALL PROBLEM COMPONENTS
C
      DO 62 I=1,NNOD
      DO 62 J=1,NNOD
   62 DSUM(I,J)=DSUM(I,J) + H(I,J) -((OMEGA**2)*Q(I,J))
      DO 63 I=1,NRBN
      II=NDAMP(I)
   63 DSUM(II,II) = DSUM(II,II) + DCMPLX(0.D0,(OMEGA*RD(I)))
      DO 64 J=2,NRBN
      I=J-1
      II=NDAMP(I)
      JJ=NDAMP(J)
      DSUM(II,JJ)=DSUM(II,JJ) + DCMPLX(0.D0,(OMEGA*RDS(I)))
   64 DSUM(JJ,II) = DSUM(JJ,II) + DCMPLX(0.D0,(OMEGA*RDS(I)))
      DO 65 J=3,NRBN,2
      I=J-2
      II=NDAMP(I)
      JJ=NDAMP(J)
      DSUM(II,JJ)=DSUM(II,JJ)+ DCMPLX(0.D0,(OMEGA*RDSS(I)))
   65 DSUM(JJ,II)=DSUM(JJ,II)+ DCMPLX(0.D0,(OMEGA*RDSS(I)))
C
C     MULTIPLICATION OF THE FORCING FUNCTION VECTOR B BY OMEGA SQUARED
C     AND STORING IN COMPLEX VECTOR DB
```

```
C
      DO 66 I=1,NNOD
      DB(I) = DCMPLX(B(I)*(OMEGA**2),0.DO)
   66 CONTINUE
C
C     SOLVING PROBLEM USING SUBROUTINE CSIMEQ
C
      CALL CSIMEQ (DSUM,DB,NNOD)
      WRITE(6,610)
  610 FORMAT('0','ABSOLUTE VALUE AND PHASE ANGLE OF PRESSURE')
      WRITE(6,611)
  611 FORMAT('0','(PRESSURE ON TOP-PHASE ANGLE ON BOTTOM)')
      DO 72 I=1,NNODT
      ABPRES(I)=0.DO
   72 PHASE(I)=0.DO
      K=0
      DO 68 I=1,NNODT
      DO 69 J=1,NE
      JJ=NN9(J,9)
      IF(JJ.EQ.I) GO TO 68
   69 CONTINUE
      K=K+1
      ABPRES(I)=CDABS(DB(K))
      A=DB(K)
      AA=DB(K)*(0.DO,-1.DO)
      PHASE(I)= 57.2957795131DO * DATAN2(AA,A)
   68 CONTINUE
      I=1
      J=9
  615 WRITE(6,612)(ABPRES(K),K=I,J)
  612 FORMAT(///9(5X,F9.5))
      WRITE(6,613)(PHASE(K),K=I,J)
  613 FORMAT(9(5X,F9.3))
      I=I+9
      J=J+9
      IF(J.LT.82) GO TO 615
C
C     ACOUSTIC IMPEDANCE AND ENERGY BALANCE
C
      WRITE(6,801)
  801 FORMAT('0','ACOUSTIC IMPEDANCE')
      Z=0.DO
      ZZ=0.DO
      DO 80 J=1,NP
      JJ=NPN(J)
      Z=Z+B(JJ)*(DB(JJ)*(0.DO,-1.DO))
   80 ZZ=ZZ+B(JJ)*DB(JJ)
      ZREAL=-2.DO*3.141592653600*Z/(OMEGA*FLDEN)
```

```
      ZIMAG=2.D0*3.1415926536D0*ZZ/(OMEGA*FLDEN)
      TENGI=-Z*OMEGA/(2.D0*FLDEN)
      WRITE(6,803) ZREAL
  803 FORMAT('0',5X,'REAL=',F16.8)
      WRITE(6,804)ZIMAG
  804 FORMAT('0',5X,'IMAG=',F16.8)
      WRITE(6,814)
  814 FORMAT('0','PROBLEM ENERGY BALANCE')
      WRITE(6,810)TENGI
  810 FORMAT('0',5X,'AVG POWER IN=',F16.8)
      ENGO=0.D0
      DO 84 I=1,NRBN
      II=NDAMP(I)
   84 ENGO=ENGO+CDABS(DB(II))*RD(I)*CDABS(DB(II))
      DO 86 I=2,NRBN
      J=I-1
      JJ=NDAMP(J)
      II=NDAMP(I)
   86 ENGO=ENGO+CDABS(DB(II))*RDS(J)*CDABS(DB(JJ))*2.D0
      DO 88 I=3,NRBN,2
      J=I-2
      JJ=NDAMP(J)
      II=NDAMP(I)
   88 ENGO=ENGO+CDABS(DB(II))*RDSS(J)*CDABS(DB(JJ))*2.D0
      TENGO=ENGO/(2.D0*SOUND*FLDEN)
      WRITE(6,812) TENGO
  812 FORMAT('0',5X,'AVG POWER OUT=',F15.8)
   70 OMEGA = OMEGA + OINCR
      RETURN
      END


      SUBROUTINE NODE(NXDIV,NYDIV)
C
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      COMMON DSUM(81,81),DB(81)
      COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
      COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XXD(8),YYD(8),AJ(2,2),BL(3,3)
      COMMON SOUND,FLDEN,OMEGA,OINCR
      COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC
C
C
C     ASSIGNING NODE NUMBERS TO ELEMENTS
C
      WRITE(6,126)
```

```
126 FORMAT('0','ASSIGNMENT OF NODE NUMBERS TO ELEMENTS')
    I1=1
    I2=NXDIV
    L=4*NXDIV+3
    K=2*NXDIV+2
    J=1
    DO 32 KK=1,NYDIV
    DO 30 I=I1,I2
    NN9(I,1)=L
    NN9(I,2)=L+1
    NN9(I,3)=L+2
    NN9(I,4)=K+2
    NN9(I,5)=J+2
    NN9(I,6)=J+1
    NN9(I,7)=J
    NN9(I,8)=K
    NN9(I,9)=K+1
    L=L+2
    K=K+2
 30 J=J+2
    L=L+2*NXDIV+2
    J=J+2*NXDIV+2
    K=K+2*NXDIV+2
    I1=I1+NXDIV
 32 I2=I2+NXDIV
    WRITE(6,127)
127 FORMAT('0','ELEMENT NO',5X,'NODE NUMBERS (8 NODED ELEMENTS)')
    DO 31 I=1,NNODT
 31 NOW(I)=0
    II=0
    DO 40 I=1,NNODT
    DO 42 J=1,NE
    JJ=NN9(J,9)
    IF(JJ.EQ.I) GO TO 40
 42 CONTINUE
    II=II+1
    NOW(I)=II
 40 CONTINUE
    DO 34 I=1,NE
    DO 35 J=1,M
    II=NN9(I,J)
 35 NN(I,J)=NOW(II)
 34 WRITE(6,128)I,(NN(I,J),J=1,M)
128 FORMAT(3X,I2,11X,8I5)
    RETURN
    END
```

```
      SUBROUTINE XYFORM(NXDIV,NYDIV)
C
      IMPLICIT COMPLEX*16 (D),REAL*8 (A-C,E-H,O-Z)
      REAL*8 ITITLE
      REAL*8 LABEL/8H        /
      COMMON DSUM(81,81),DB(81)
      COMMON H(81,81),Q(81,81),B(81),X(81),Y(81),HE(8,8),QE(8,8)
      COMMON RD(50),RDS(50),RDSS(50),XM(8),YM(8),AMP(25)
      COMMON GA(4),GH(4),SF(8),XD(8),YD(8),XXD(8),YYD(8),AJ(2,2),BL(3,3)
      COMMON SOUND,FLDEN,OMEGA,OINCR
      COMMON NN(100,8),NDAMP(50),NPN(50),NSF(8),NG(8),NN9(50,9),NOW(100)
      COMMON NE,M,NNOD,NRBN,NP,NO,NNODT,NC
      REAL*4 XDRAW,YDRAW
      COMMON /TITLE/ITITLE(12)
      DIMENSION XVAL(25),YVAL(25),XDRAW(25),YDRAW(25),XF(100),YF(100)
C
C
C     CALCULATE X AND Y COORDINATES
C
      XINCR=1.DO/NXDIV
      YINCR=1.DO/NYDIV
      NX=2*NXDIV+1
      NY=2*NYDIV+1
      DO 10 I=1,NX
   10 XVAL(I)=0.DO
      DO 11 I=1,NY
   11 YVAL(I)=0.DO
      XVAL(1)=-1.DO
      DO 12 I=2,NX
      J=I-1
   12 XVAL(I)=XVAL(J)+XINCR
      YVAL(1)=1.DO
      DO 19 I=2,NY
      J=I-1
   19 YVAL(I)=YVAL(J)-YINCR
      WRITE(6,121)
  121 FORMAT('0','VALUES OF XI AND ETA FOR WHICH X AND Y COORDINATES WIL
     1L BE CALCULATED')
      WRITE(6,126)
  126 FORMAT('0',5X,'VALUES OF XI',5X,'VALUES OF ETA')
      WRITE(6,125)(I,XVAL(I),YVAL(I),I=1,NX)
  125 FORMAT(I2,5X,F10.5,5X,F10.5)
      LL=1
      DO 16 I=1,NY
      DO 27 J=1,NX
      GX=XVAL(J)
      GY=YVAL(I)
      SF(1)= .25DO*(1.DO-GX)*(1.DO-GY)*(-GX-GY-1.DO)
```

```
      SF(2)= .50D0*(1.D0-GX**2)*(1.D0-GY)
      SF(3)= .25D0*(1.D0+GX)*(1.D0-GY)*(+GX-GY-1.D0)
      SF(4)= .50D0*(1.D0+GX)*(1.D0-GY**2)
      SF(5)= .25D0*(1.D0+GX)*(1.D0+GY)*(+GX+GY-1.D0)
      SF(6)= .50D0*(1.D0-GX**2)*(1.D0+GY)
      SF(7)= .25D0*(1.D0-GX)*(1.D0+GY)*(-GX+GY-1.D0)
      SF(8)= .50D0*(1.D0-GX)*(1.D0-GY**2)
      XF(LL)=0.D0
      YF(LL)=0.D0
      DO 28 K=1,M
      XF(LL)=XF(LL)+SF(K)*XM(K)
   28 YF(LL)=YF(LL)+SF(K)*YM(K)
   27 LL=LL+1
   16 CONTINUE
      WRITE(6,127)
  127 FORMAT('0','VALUES OF X AND Y')
      WRITE(6,120)
  120 FORMAT('0','(X COORDINATES ON TOP-Y COORDINATES ON BOTTOM)')
      I=1
      J=2*NXDIV+1
      N=(2*NXDIV+1)*(2*NYDIV+1)
      WRITE(6,128)(L,L=I,J)
  128 FORMAT('0',8X,I2,8(11X,I2))
  124 WRITE(6,122)(XF(L),L=I,J)
  122 FORMAT('0',9(5X,F8.5))
      WRITE(6,123)(YF(L),L=I,J)
  123 FORMAT(9(5X,F8.5))
      I=I+(2*NXDIV+1)
      J=J+(2*NXDIV+1)
      IF(J.LT.(N+1)) GO TO 124
C     PLOT CORNER NODES AS POINTS ON A GRAPH
C
      I=1
      J=2*NXDIV+1
      II=1
      JJ=(2*NXDIV+1)*(2*NYDIV)+1
      NNX=NXDIV+1
      NNY=NYDIV+1
      KK=4*NXDIV+2
  129 LL=1
      DO 15 L=I,J,2
      XDRAW(LL)=XF(L)
      YDRAW(LL)=YF(L)
   15 LL=LL+1
      K=2
      IF(I.EQ.1) K=1
      CALL DRAW(NNX,XDRAW,YDRAW,K,0,LABEL,ITITLE,0,0,0,0,0,0,5,7,0,LAST)
      I=I+(4*NXDIV+2)
```

```
      J=J+(4*NXDIV+2)
      IF(J.LT.(N+1)) GO TO 129
   18 LL=1
      DO 17 L=II,JJ,KK
      XDRAW(LL)=XF(L)
      YDRAW(LL)=YF(L)
   17 LL=LL+1
      K=2
      IF(JJ.EQ.N) K=3
      CALL DRAW(NNY,XDRAW,YDRAW,K,0,LABEL,ITITLE,0,0,0,0,0,0,5,7,0,LAST)
      II=II+2
      JJ=JJ+2
      IF(JJ.LT.(N+1)) GO TO 18
      DO 34 I=1,NNOD
      X(I)=0.D0
   34 Y(I)=0.D0
      II=0
      DO 30 I=1,NNODT
      DO 32 J=1,NE
      JJ=NN9(J,9)
      IF(JJ.EQ.I) GO TO 30
   32 CONTINUE
      II=II+1
      X(II)=XF(I)
      Y(II)=YF(I)
   30 CONTINUE
      RETURN
      END


      SUBROUTINE CSIMEQ (A,B,N)
C
C     CSIMEQ IS A SUBROUTINE TO SOLVE EITHER A REAL OR COMPLEX SYSTEM
C     OF SIMULTANEOUS EQUATIONS IN WHICH MATRIX A IS ANY SQUARE MATRIX
C     AND B IS THE COLUMN VECTOR OF CONSTANTS ON THE RIGHT SIDE OF THE
C     EQUATIONS.  N IS THE NUMBER OF EQUATIONS IN THE SYSTEM
C
      COMPLEX*16 A(81,81),B(81)
C
      DO 401 I=1,N
      I1=I+1
      B(I)=B(I)/A(I,I)
      IF (I-N) 402,403,402
  402 DO 404 J=I1,N
      IF(CDABS(A(I,J)) .EQ. 0.D0) GO TO 404
      A(I,J)=A(I,J)/A(I,I)
      DO 406 K=J,N
      A(K,J)=A(K,J)-A(K,I)*A(I,J)
```

```
  406 A(J,K)=A(K,J)
  404 B(J)=B(J)-A(J,I)*B(I)
  401 CONTINUE
C
C     BACK SUBSTITUTION
C
  403 I1=I
      I=I-1
      IF(I) 411,411,412
  412 DO 413 J=I1,N
  413 B(I)=B(I)-A(I,J)*B(J)
      GO TO 403
  411 RETURN
      END
```

APPENDIX C

ISO-PARAMETRIC ELEMENTS

The family of iso-parametric elements, recently developed by Zienkiewicz, Irons, and others, has brought new versatility and power to the finite element method. The special usefulness of these elements is that they may be shaped to conform to curvilinear boundaries. Attention is confined here to two-dimensional parabolic elements. A full description of the entire family is given in Ref. [2].

Consider the rectangular element of Figure 5. The element has four corner nodes and four midside nodes, numbered as shown. Points in the interior, or on the boundary, may be located in the local (ξ,η) coordinate system. The dependent variable, pressure p in the present application, is defined at any point by Eq. (3), repeated here

$$p = \sum_{i=1}^{8} N_i p_i \tag{3}$$

where

$$N_i = \frac{1}{4}(1 + \eta\eta_i)(1 + \xi\xi_i)(\eta\eta_i + \xi\xi_i - 1)$$

for corner node i, and

$$N_i = \frac{1}{2}(1 - \eta^2\xi_i{}^2 - \xi^2\eta_i{}^2)(1 + \eta\eta_i + \xi\xi_i)$$

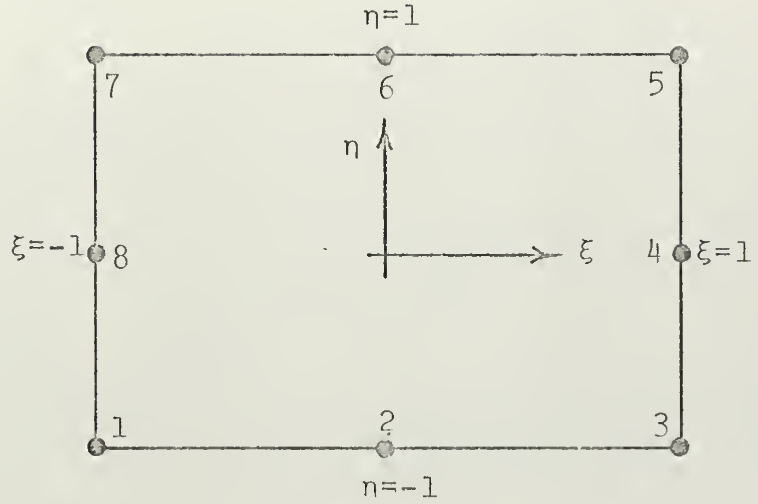

Figure 5. Element Configuration in Local ξ,η Coordinates

for mid-side node i,

(ξ_i, η_i) are coordinates of node i,

and p_i is the pressure at node i.

Note that each shape function N_i has the value unity at node i, and is zero at all other nodes.

A curvilinear element, such as that of Figure 6, may be produced by mapping the rectangle of Figure 5 into the x,y plane by the relations

$$x = \sum_{i=1}^{8} N_i x_i, \qquad y = \sum_{i=1}^{8} N_i y_i \tag{19}$$

where (x_i, y_i) are the cartesian coordinates of node i.

The shape of the element in the x,y plane is uniquely determined by specifying the coordinates (x_i, y_i) for the

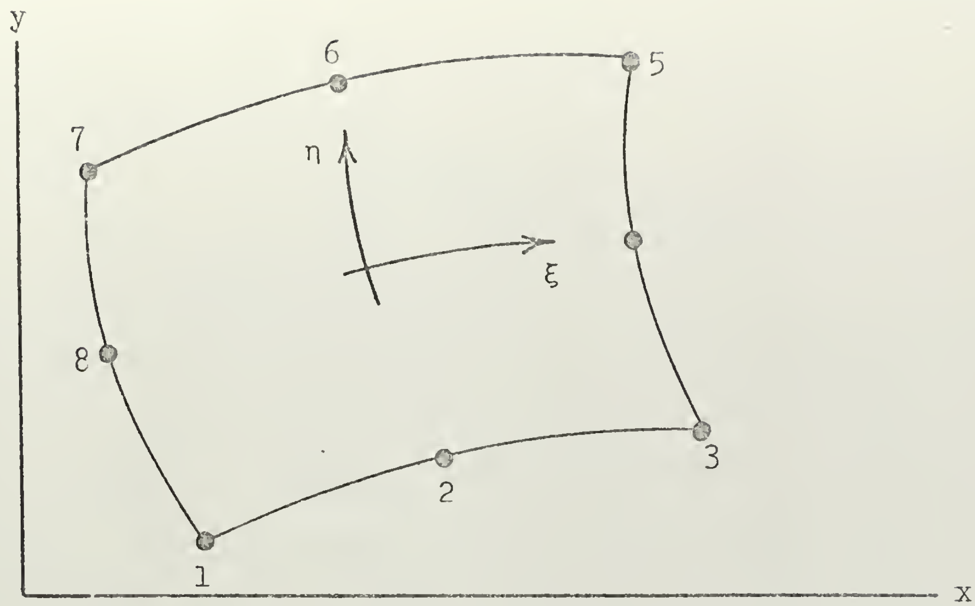

Figure 6. Curvilinear Elements Mapped in Cartesian Coordinates.

eight nodes. The edges of this curvilinear element are parabolic.

Adjacent elements "share" the same three nodes at their interface. When mapped to cartesian coordinates, these two adjacent elements then have the same curvilinear shape and the same x and y coordinates at nodes. This provides continuity in the pressure distribution across element boundaries.

The region developed for the present study necessitates the use of iso-parametric elements to provide the circular non-reflection boundary. This boundary is constructed using parabolic arc segments which are mapped by use of the shape functions to "fit" a quarter circle boundary in cartesian coordinates.

APPENDIX D

FINITE ELEMENT MESH GENERATOR

Two subroutines, included in the computer program of Appendix B, should prove useful in a variety of finite element problems. Together, these subroutines subdivide the region, systematically assign element numbers and node numbers, calculate nodal coordinates, and produce a computer plot of the region to be studied. As an example, consider the region of Figure 7.

The region is taken to be one large iso-parametric element (a mapping of the rectangle of Figure 5). Its shape is prescribed by specifying the (x,y) coordinates of the

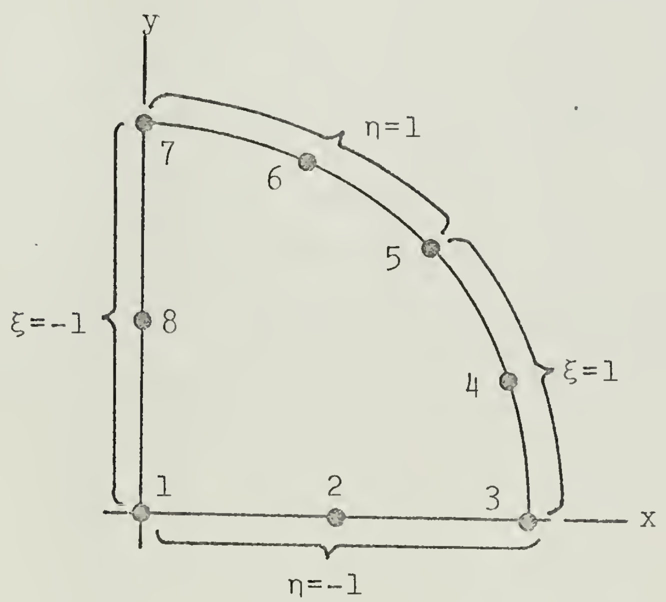

Figure 7. Basic Geometry of Finite Element Region.

eight nodes. Subdivision of the element can then be accomplished at predetermined values of ξ and η by using Equation (19).

Subroutine NODE (Appendix B) requires two input parameters, the number of divisions desired in the x direction and the number of divisions desired in the y direction. These parameters are used to calculate the increment values of ξ and η for subdividing the element into smaller elements. The subroutine starts numbering global nodes at the upper left hand corner ($\xi=-1$, $\eta=1$) then adds increment values of ξ and assigns global node numbers from left to right on the boundary $\eta=1$. Similarly, after incrementing η, the second row of global node numbers is assigned, etc. Global node numbers are assigned to all corner nodes, mid-side nodes and the center of each subdivided element (9 nodes per element). The center node provides unique identification for the element and simplifies the printing of computer output in a readable form. Subsequently all nodes are renumbered, omitting the center nodes.

The cartesian coordinates corresponding to the global node numbers are calculated by a similar process in the subroutine XYFORM and are used in the program to plot the finite element grid. Figure 8 illustrates a computer plot of the finite element mesh generated for a quarter circular region (Figure 7) with an eight foot radius. The number of subdivisions in both the x and y direction is equal to four. The plotting routine produces straight segments

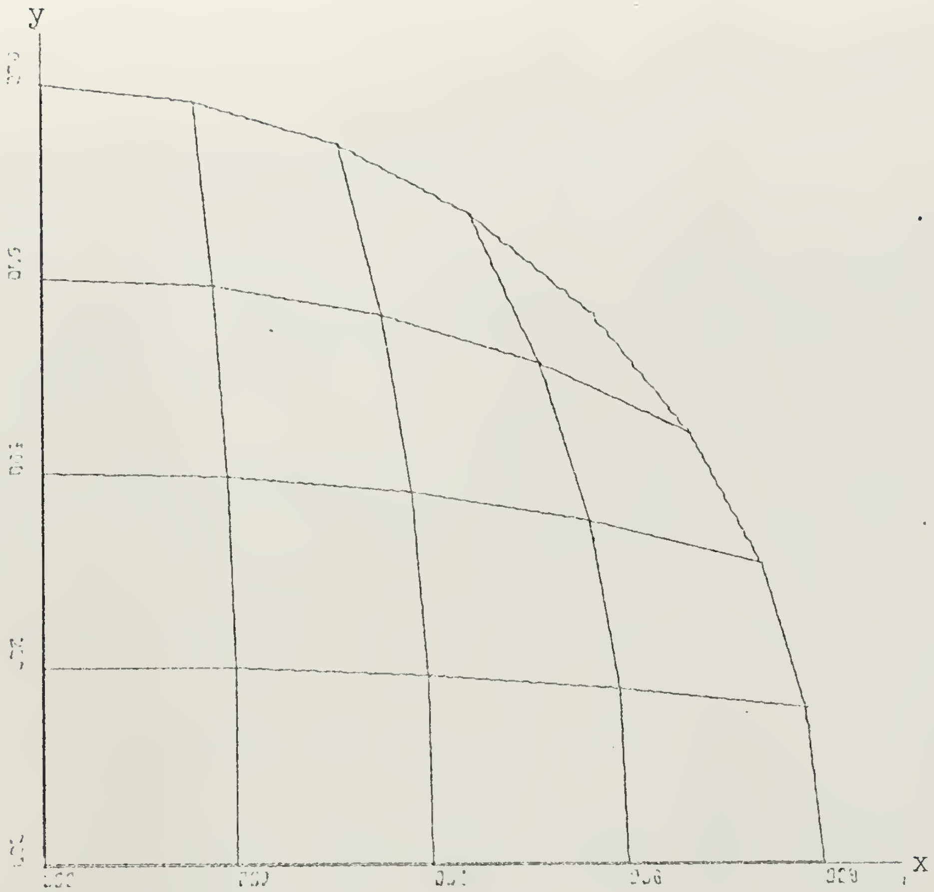

Figure 8. Finite Element Mesh Generated by Subroutine XYFORM.

between nodes. In this instance, the element boundaries are all parabolic arcs except for those lying on the x and y axes.

APPENDIX E

MATRIX ELEMENT FORMULAS

Individual elements of the matrices [Q], [D], [L], and [H] of the system of ordinary differential equations (Eq. (4)) are given by Ref. [3] as

$$q_{ij} = \frac{1}{c^2} \int_{Re} N_i N_j \, dR$$

$$d_{ij} = \frac{1}{c} \int_{Se} N_i N_j \, dS$$

$$\ell_{ij} = \int_{Se} N_i' N_j \, dS$$

$$h_{ij} = \iint_{Re} \left[\frac{\partial N_i}{\partial x} \cdot \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \cdot \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \cdot \frac{\partial N_j}{\partial z} \right] dR$$

where Re and Se are the element region and external boundary respectively.

The shape functions N_i, N_i' are specified in terms of local coordinates and it is advantageous to perform the integrations in these coordinates. Gaussian quadrature is employed. Details of the procedure are given in Ref. [2].

LIST OF REFERENCES

1. Hunter, Joseph L., Acoustics, Prentice-Hall, 1957.

2. Zienkiewicz, O. C., Irons, B. M., et al., "Iso-Parametric and Associated Element Families for Two-and Three-Dimensional Analysis," Chapter 13 in Finite Element Methods in Stress Analysis, Holand, I. and Bell, K. (eds.), pp. 403-416, Tapir, Trondhiem, 1969.

3. Zienkiewicz, O. C. and Newton, R. E., "Coupled Vibrations of a Structure in a Compressible Fluid," Proceedings of the Symposium on Finite Element Techniques. University of Stuttgart, Germany, June 1969.

4. Kinsler, L. E., and Frey, A. R., Fundamentals of Acoustics, 2nd ed., John Wiley & Sons, 1962.

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13. ABSTRACT

The generation and propagation of small amplitude acoustic waves in a homogeneous, loss-free, compressible fluid is studied by the finite element method. A diaphragm mounted in an infinite rigid baffle generates acoustic waves in a semi-infinite fluid region. Steady-state pressure distribution is found for a hemispherical region with boundary reflection suppressed through use of a radiation condition. The computer program developed for the purpose utilizes iso-parametric finite elements with curvilinear boundaries. Incorporated in the program is a versatile mesh generator which minimizes the quantity of input data. Acceptable agreement with analytic results is obtained when there are at least four elements per wave-length.

14 KEY WORDS	LINK A		LINK B		LINK C	
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